Why we study physics

For the Christian, the motivation for studying the world around us is quite clear: the creation reflects the nature of the creator. Because man is created in the image of God (Genesis 1:26), we are able to both understand and appreciate the beauty of the world around us.

What is physics? In physics, one learns the *basic principles* that govern all natural phenomena. Thus, we try to answer basic questions like:

- What does it mean to make a measurement? How "accurate" is a given measurement?
- How do objects move?
- How do objects interact?
- What quantities are conserved?
- How do waves describe things?
- What is heat?

All of the natural sciences: astronomy, biology, chemistry, geology, *et cetera* rest on the basic principles that are studied in physics.

> The heavens declare the glory of God;
> the skies proclaim the work of his hands.
> Day after day they pour forth speech;
> night after night they display knowledge.
> There is no speech or language
> where their voice is not heard.
> Their voice goes out into all the earth,
> their words to the ends of the world.
>
> Psalm 19:1–4

Introduction

Here are some general directions for the weekly laboratory part of Physics 181 and 201.

Materials For each meeting of the laboratory, you should bring:

- This lab manual;
- Your laboratory notebook, as discussed below;
- A pen for writing in your laboratory notebook; Don't use pencil for this!
- A calculator;
- A physics textbook for looking up formulas and constants.

Grading Each lab's work is graded for ten points. The lab reports are due at the beginning of the next lab. The laboratory notebook will be collected and graded for ten points at the end of the course. The total for all the reports and the laboratory notebook will be reported to your lecture instructor at the end of the term.

Attendance Attendance is required for all laboratory activities. If you know of a conflict for a future meeting, arrange with the instructor to attend another section of the lab for that week. Any work to make up for an excused absence is to be determined by the instructor.

Laboratory Notebook You should bring to each meeting of the laboratory your laboratory notebook. The notebook must be a bound, quadrille (graph paper) book. If you have such a notebook from a previous laboratory with at least fifty blank pages, this should suffice for one semester of lab work. The first three pages should be reserved to enter your name, the course title, and a table of contents to be completed by the end of the semester. The entries for each session should begin on a new page, which begins with the title for the work. All observations and data are to be written in ink directly into the record as they are made—not copied from scraps of paper—into the notebook. Any measurement must include the appropriate units. If data are discarded, they are simply crossed out with a single $\times$ so that the stricken material is still visible. The last page on which data are recorded for the day should be signed and dated, then signed by the instructor before you leave for the day.

Reports Laboratory activities are to be submitted at the following meeting as a formal report. Work is which is submitted later than the due date will be assessed a penalty. Your report should be viewed as an opportunity to practice your writing skills. It should include the following sections:

Abstract Briefly describe the experiment and summarize your main results.

Procedure Write a short summary of the experimental set-up and the measurements performed.

Data Present your data in well organized form. Do not include large amounts of raw data if you include a graph or statistical analysis of that data.

Analysis Perform any calculations needed. This should be done first in your lab notebook and—when you get everything right—copied into your report. You may find it convenient to write this part of the report by hand.

Conclusions Summarize your most important results. When appropriate, compare them with the standard values and discuss any discrepancies. The conclusion should be quantitative and discuss differences in terms of standard deviations.

Be sure to include the names of your lab partners at the beginning of your report. Reports may be typed or neatly hand-written. Unless otherwise instructed, the first four sections of your report may be done as a group; however, each student must write his own conclusion.

To use your time efficiently in the laboratory, you should read the appropriate section of this lab manual before coming to lab. Think of questions to ask at the beginning of the session concerning procedures which are not clear. Look for questions in the lab manual which you will need to answer in your report.

Each member of the group is to take part in the work. The group is to share in all aspects of the work, including the calculation and analysis phase.

In grading your reports, your instructor will have several points in mind: Did this person participate and contribute in the lab phase of the work? Is the report well organized; is it easy to find the main items? Can the main result of the experiment be found quickly? Have all the parts of the experiment been treated and included? Is the writing reasonably free of grammatical and other writing errors? Was an error overlooked because data analysis was not begun before leaving the lab?

Physics 181/201 Schedule 2003

Experiment	Monday	Tuesday	Thursday	Friday
1	Sept. 8	Sept. 9	Sept. 4	Sept. 5
2	Sept. 15	Sept. 16	Sept. 11	Sept. 12
3	Sept. 22	Sept. 23	Sept. 18	Sept. 19
4	Sept. 29	Sept. 30	Sept. 25	Sept. 26
5	Oct. 6	Oct. 7	Oct. 2	Oct. 3
6	Oct. 13	Oct. 14	Oct. 9	Oct. 10
7	Oct. 20	Oct. 21	Oct. 16	Oct. 17
8	Nov. 3	Nov. 4	Oct. 30	Oct. 31
9	Nov. 10	Nov. 11	Nov. 6	Nov. 7
10	Nov. 17	Nov. 18	Nov. 13	Nov. 14
11	Nov. 24	Nov. 25	Nov. 20	Nov. 21
12	Dec. 1	Dec. 2	Dec. 4	Dec. 5

All laboratory notebooks and reports are due at 1:30 PM one week after your last lab.

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Experiment 1

Measurements, Units, and Errors

The objectives for the first session are:

1. to introduce the concept of experimental measurement,
2. to develop the use of dimensions, SI units, and unit conversion,
3. to examine sources of experimental error and tools for treating it, and
4. to treat errors in a simple measurement system.

The bulk of this lab consists of a discussion of measurement. The final activity for the session will involve some measurements to illustrate these concepts. The data should be recorded in your lab notebook as described in the Introduction. In your textbook you should locate any corresponding material on measurement and units.

There are two basic kinds of measurements that are performed in physics experiments. The first kind is the "counting" measurement where the number of electrons, photons, or other particles is counted. The most sensitive and precise measurements tend to be of this kind. The second kind is the measurement of a "continuous quantity" such as mass, length, electrical voltage, and the like. This kind of measurement will be the focus of this course. Any measurement has a *value*, an *uncertainty* (error), and a *dimension*.

1.1 Dimensions

Measurement is a comparison between a *standard* and a *quantity* of the same *dimension*. For example: I use a ruler (the standard) to measure the length of a rod (the quantity). In this case, the dimension is length. Other examples of dimensions are: mass, time, velocity, force, charge, *et cetera*. The standard chosen for a dimension is called its unit. The preferred choice for units in physics is the set adopted by the international community of scientists, called the SI system, for "Systéme Internationale d'Unités." Occasionally, we will use other units: inches, grams, and so forth, when we find it convenient to do so.

The dimensions in physics are interrelated. In mechanics, the primary dimensions are: *mass*, *length*, and *time*. Many other quantities—velocity, force, viscosity, *et cetera*—have dimensions which are composites of the primary dimensions. For example, the dimension *velocity* can be expressed as *length*/*time*.

The SI system assigns units defined by international standards to the primary dimensions; standards for the secondary dimensions are then determined from the primary standards. The SI system assigns the kilogram (kg) to mass; the meter (m) to length; and the second (s) to time. The system suffers from its genealogy and history, as seen in the choice of the kilogram. One kilogram equals 1000 grams; the gram was the old standard unit. The kilogram was chosen as the mass unit because a standard of that size could (and still can) be measured more precisely. It would have made more sense to choose a new name for the kilogram, allowing a single letter for its symbol. But with the system we have inherited, it would be awkward to use the millikilogram (= 1 gram). The prefixes for mass have been applied instead to the gram: milligram, microgram, etc.

Units are useful for checking calculations. The units on the left hand side of any equation must match the units on the right hand side. For example, one can only compare length with length, mass with mass, *et cetera*. In addition, one can only add like quantities, *id est* those sharing the same dimensions. Be sure to include units for all measurements recorded in your lab notebook. Your current instructor is a units fanatic, so it behooves you to *get them right.*

The disparity among systems of units often requires one to convert a measurement from one set of units to another. This conversion is conveniently carried out using the unit conversion process, which is assumed to be familiar to the reader and is covered in any physics textbook. Note that the dimensions stay the same under a units conversion.

1.2 Experimental Uncertainty and Errors

Every measurement involves uncertainty. When measuring continuous quantities, there is always some minimum change in the quantity below which the measuring instrument will not respond. These experimental uncertainties are customarily called errors. The use of the term "errors" does not imply that anyone has made a mistake. The gross error of simply misreading an instrument is called a "blunder." We are concerned with two categories of error: systematic error, the relative absence of which we call accuracy or closeness to the true value; and statistical error, the relative absence of which we call precision or reproducibility.

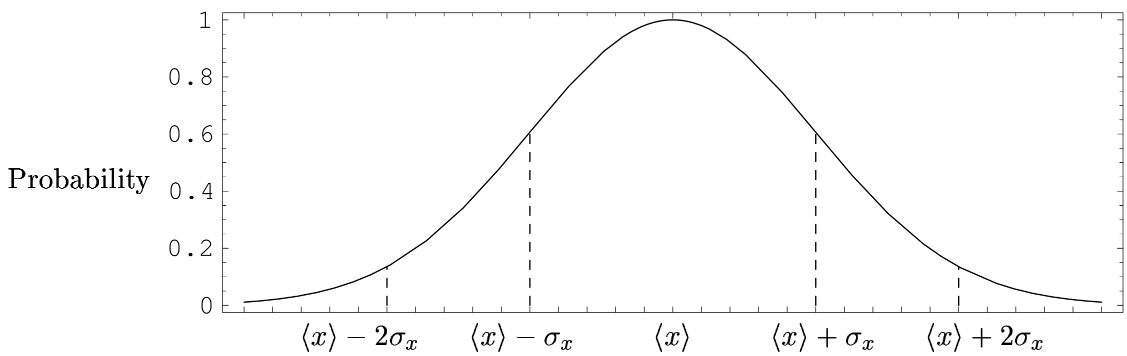
Systematic error, as its name implies, usually manifests itself in repeated measurements which are always higher—or always lower—then they should be. This type of error is usually associated with a defect in the measuring system or a lack of calibration. A clock which runs slow always measures the time interval for some process as smaller than the time interval reported by the official time standard maintained by the National Institute of Standards. Another example would be a ruler whose end has been worn down; all measurements made with that end of the ruler will be too small by some amount.

Statistical error occurs when the result varies from measurement to measurement. The fluctuations are on the high side as well as on the low side. Such fluctuations are often found to follow probability models. The most widely applicable model is the Gaussian distribution. When this model is applied to a set of repeated measurements, estimates can be made as to the relative likelihood that an error of a certain magnitude will occur in a single measurement. More importantly, the model provides a range of values within which the correct result is expected to be found with some level of confidence.

One defines, for a set of n repeated measurements of quantity x, a mean value $\langle x \rangle$ and a standard deviation σ_x. We define x_i, $i \in \{1, 2, \ldots, n\}$, to be the result for each of the individual measurements. In equation form:

$$\langle x \rangle = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad \sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \langle x \rangle)^2}{n-1}} .$$

In words, the mean is the sum of all the measurements divided by the total number of measurements. Note that dividing the sum of the squares of the deviations by $n-1$ gives a result which diverges if there is only one measurement. This makes sense because we know nothing about the precision of a measurement if we have only one example. The standard deviation gives a quantitative estimate of the lack of precision of the set of measurements. For example, any single measurement has a probability of about 68% of not being more than one standard deviation away from the mean value (high side or low side). Likewise, we are 95% confident that a given measurement will be not more than two standard deviations different from the mean value. The graph below illustrates the Gaussian distribution.



In general, standard deviations are written as a single digit and mean values should not include digits smaller than the standard deviation. For example: $\langle x \rangle \pm \sigma_x = 10.2 \pm 0.3$ cm.

Suppose that the result of an experiment is 10.2 ± 0.3 cm. This implies that the chances are 2 out of 3 that the next measurement will be in the range of 9.9 cm to 10.5 cm. Consider two competing theories: theory A and theory B. Theory A predicts a result of 9.2 cm, while Theory B predicts a result of 10.4 cm. We conclude that B's prediction agrees with observation to within experimental error, but A's prediction does not. Now, suppose theory A and theory B predicted values of 9.9 cm and 10.3 cm; we would have concluded that our experiment cannot distinguish between them, to within experimental error. A more precise measurement would be needed.

Reporting experimental results correctly with appropriate error analysis may seem to be an imposition of unnecessary work, but honest presentation of experimental results is one of the basic ethical principles of science. Without this honesty, science as it is carried out today would be virtually impossible.

An Example

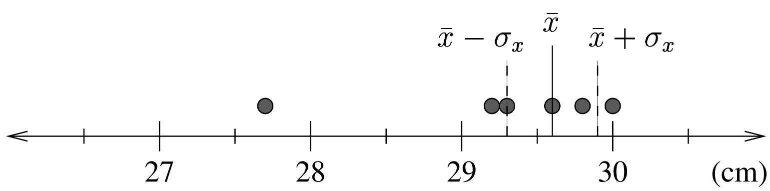
The following set of six values represents successive measurements of a length (in cm):

29.2 29.8 29.6 29.3 30.0 27.7

We suspect that the last of these is a blunder; we will start by finding the mean and standard deviation

$$
\begin{aligned}
\langle x \rangle &= \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{29.2 + 29.8 + 29.6 + 29.3 + 30.0 + 27.7}{6}\,\text{cm} = 29.27\,\text{cm} \\
\sigma_x &= \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \langle x \rangle\right)^2}{n-1}} \\
&= \sqrt{\frac{(29.2\,\text{cm} - 29.27\,\text{cm})^2 + (29.8\,\text{cm} - 29.27\,\text{cm})^2 + \cdots + (27.7\,\text{cm} - 29.27\,\text{cm})^2}{6-1}} \\
&= 0.8238\,\text{cm}
\end{aligned}
$$

Thus, the average value is $x = 29.3 \pm 0.8\,\text{cm}$. We see that the data point 27.7 is about $2\sigma_x$ from the mean. This suggests that the questionable data point may indeed be a blunder. We then proceed to recalculate $\langle x \rangle$ and σ_x, omitting the questionable value. The student should verify that, without this point, $\langle x \rangle = 29.58\,\text{cm}$ and $\sigma_x = 0.33\,\text{cm}$. The difference between the datum 27.7 cm and the recalculated mean is 1.88 cm, which is almost six times as large as the new standard deviation. When a value exceeds three or four times the standard deviation established by the other members of the set, it is probably better to reject it.



In this case, we would quote the result as $x = 29.6 \pm 0.3\,\text{cm}$. If "outliers" such as this have been removed from the data, this fact must be mentioned in any analysis of the data.

Relative Standard Deviation

Another quantity, which is related to the standard deviation, is frequently more useful for error calculations. The relative standard deviation r is the ratio of the standard deviation to the mean value: $r_x = \sigma_x / \langle x \rangle$. For the result $10.3 \pm 0.4\,\text{cm}$, the relative standard deviation is $r = 0.4/10.3 = 0.04$ or 4%; likewise, for $29.6 \pm 0.3\,\text{cm}$, $r = 0.3/29.6 = 0.01$ or 1%. The percentage form will be frequently used in expressing relative precisions of results in your reports. Note that r does not have any dimensions.

A frequent comment from students is that we could simply eliminate all errors if only Geneva College would purchase better lab equipment. However,

- better equipment (if actually available) makes the error smaller, but never eliminates it; and
- increased precision is expensive.

To improve a simple measurement by one significant figure, the standard deviation has to be made one tenth as large. Typically this requires equipment which costs 100 times as much and may require months of measurement instead of minutes. The art of measurement is to procure just enough precision for the objective. Knowing how much is enough is not always simple. In our next session we will examine a method, known as propagation of errors, which permits us to estimate the effect of one type of measurement error on the overall result of a process which requires several kinds of measurement taken together.

1.3 Computer Analysis

Some of the measurements and analyses in this lab will be performed using a computer. The computers in the lab use a "UNIX" type operating system and an "X-windows" graphical user interface. Here are a few general guidelines:

- If your computer shows a login screen, supply the user name `luser` and password `geneva`.
- Double-click on the `physics` icon to start up the physics program.
- When plotting, you can remove unwanted plots by placing the cursor over the plot and typing the letter `q`.
- The XTerm windows have a scrolling option.
- Do not turn the computers off.

For today's work, start the `physics` program and select the menu item `Math`. One of the tools in the `Math` menu is a standard deviation calculator. With your lab group, use one of the computers to enter the data of the above example. Try another sequence of numbers of your own. In the short experiment your group will perform today, means and standard deviations will be required.

1.4 Pendulum data for error analysis

1. Have each member of the team independently measure the length of the pendulum L with a meter stick. The length is estimated as the distance from the bottom of the clamp, where the string bends, to the center of the pendulum bob. Don't tell your lab partners the result of your measurement! This could prejudice the results that they obtain. Repeat until there are a total of at least ten measurements for the lab group.

2. Because one must estimate the position of the center of the bob, each observer will obtain slightly different values. Now, compare your results. Discarding any "blunders," determine the mean and standard deviation for the length using the ten values obtained above.

3. Set the pendulum in motion with an amplitude of 2 ± 0.5 in., that is, about 4 in. between the extremes of motion. Allowing the pendulum to continue swinging, use the stopwatch to measure the time for one complete cycle or "period."

   Have each team member perform five measurements, allowing the pendulum to continue swinging between measurements. If the pendulum swings become much smaller than the 2 in. amplitude, restart the motion with the 2 in. amplitude. Record the five values of the period T in seconds. Again, don't share your results with your team members until everyone is done.

4. Compare the results of all of the team observers. Have each observer calculate a mean value and standard deviation for his or her set of five values. Calculate a mean and standard deviation for the results of all the team members.

5. Have each team member measure the period three times in the following manner: Start the pendulum with the 2 in. amplitude and use the stopwatch to measure the total time for the bob to start at the right extreme then return to the right extreme ten times. Divide this total time by ten to obtain the time for one period.

6. When everyone is done, find the mean and standard deviation for these results. Is the standard deviation for the measurements of the period smaller using this method? Label this period T' for later analysis.

1.5 Error Analysis

Using your mean values for L and T, calculate the acceleration of gravity g using the simple pendulum formula $g = 4\pi^2 L/T^2$. Using your estimates of the standard deviations for L and for T, find the relative standard deviations for L and for T. Which has the larger relative standard deviation? The quantity with the larger relative standard deviation contributes more to the overall error in our measurement of g. Next, we calculate the relative standard deviation for g itself. It is given by the formula:

$$\frac{\sigma_g}{g} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(2\frac{\sigma_T}{T}\right)^2}\,. \tag{1.1}$$

The source of this formula is discussed in the next lab. Once you have the relative standard deviation, multiply it by your value for g to obtain a value for σ_g. This is an estimate of error for your measurement of g. You can write your result in the standard form, *id est* $g = 9.89 \pm 0.89\,\frac{\text{m}}{\text{s}^2}$.

The accepted value of g for this location is $9.807\,\text{m/s}^2$. Compare this value to your results: Is the difference between them larger than your estimate of error, σ_g? As we learned earlier, 60% of the time, the difference should be less than σ_g and 95% of the time, the difference should be less than $2\sigma_g$. If the difference is much larger than our estimate of the error, then there was some systematic error—or an experimental blunder—that was overlooked. How well did you do?

Repeat the analysis above, using the values determined for T' in place of T; the use of multiple swings should result in a smaller standard deviation for the period measurement. The L values and standard deviation will remain the same. Has this decreased the estimated error for g? Which is now the primary source of error in g? Is it L or T'? What improvements can you suggest for improving the precision in measuring L?

1.6 Exercises

These problems should be done individually and attached to the back of the group report.

1. Use the unit conversion method to convert 30 miles per hour to meters per second. 1 mile = 5280 ft; 1 ft = 12 in.; 1 in. = 2.54 cm; 100 cm = 1 m; 1 hour = 60 min; 1 min = 60 s.

2. Convert a cgs power value of $3.1\,\mathrm{g}\cdot\mathrm{cm}^2/\mathrm{s}^3$ to the value in SI units.

3. A set of six measurements, in seconds, for a pendulum's period is reported:

   $$1.94 \quad 2.02 \quad 1.96 \quad 1.97 \quad 1.95 \quad 2.00$$

   Find the mean, the standard deviation, and the relative standard deviation for this set of measurements.

Experiment 2

Pendulum Length and Period

The objectives for this session are:

1. to introduce linear regression analysis and interpretation for laboratory data,
2. to apply a regression program to data generated with rod pendulum measurements, and
3. to examine the propagation of errors method.

An important strategy in experimental physics is to varying one parameter in an experiment—while keeping all other parameters fixed—and seeing how this affects the results of the experiment. Graphing is a powerful ways to analyze data to determine the interdependence of experimental variables. Points which do not fit the curve generated by other points can be further investigated for sources of error. Different ways of plotting variables may lead to a graph which is approximated as a straight line, the simplest curve for theoretical analysis.

When a straight line relationship is found between two quantities, statistical analysis can be used to quantify the graphical results. A "linear regression" analysis provides additional information about the quality of the data and the reliability of the results. Many of our experiments will be analyzed to develop straight line relationships, from which slope or intercept values lead to important physical quantities.

2.1 Pendulum period vs. length

The experimental measurements are an extension of the pendulum observations of the previous experiment. Instead of measuring the period for a single pendulum, the period will be measured for several pendula of different length. The data will then be used to illustrate some of the graphical and statistical tools for analyzing the interdependence of experimental variables.

You will measure the length with a metal meter stick which is accurate to within 0.5 mm. The period (time for one cycle) will be measured with a photogate timer, resolvable to 0.0001 second, removing the error of human reaction time associated with the use of the stopwatch.

Precision is also affected by parameters which vary randomly while performing the series of measurements. Two obvious sources of variation in the pendulum are the mass and the amplitude. A theoretical analysis shows that the mass of the pendulum has no effect on the period. The second parameter is the amplitude. Theory suggests that the effect of amplitude for small amplitude oscillations is that the period increases slightly with the square of the amplitude. To minimize this error, we will perform the experiment with very small amplitudes. Multiple observations of the period for each length will improve the reliability of the statistical analysis, as well as affording an opportunity to catch any major blunders.

The pendulum system to be used will consist of a steel rod with a pivot pin through one end. The theoretical equations describing a rod pendulum are slightly different from those for the simple ball pendulum. The advantage is that the rod has an easily measured length which is not effected by stretching forces. The model that we will use assumes that the pendulum is infinitely longer than it is thick and that the knife-edge mechanism has zero mass. Your observation of our real rod pendulums will demonstrate that we do not meet this model precisely, especially for the smaller pendula.

2.2 Procedure

The `physics` program menu offers `Pendulum experiments` for precision measurement of the periods. For each pendulum, record your results in a table in your lab notebook. Measure the length of each one of the set of seven pendula using the precision metal meter stick. The pendulum length will be defined as the distance between the bottom edge of the knife-edge and the bottom of the pendulum and should be read to ± 0.5 mm. Place the bearing edge of the pivot pin against the zero end of the meter stick and read the position marked by the other end of the pendulum rod. As a refinement, you may want to check the measurement using both sides of the pivot pin, averaging the length measured using the two sides. At least three measurements should be taken for each length, recording the average of the set.

The formula we will use to analyze our data is valid in the limit of small amplitude oscillations. Thus, you want to keep the amplitude small, less than 1 in. Repeat the measurement of the period a few times to make sure that your results are reproducible. Proceed through a set of seven different rod pendula; the order of the samples is of no consequence.

2.3 Data Analysis

We will use a linear regression to analyze the data; this will allow us to see how length affects the period. The `Math` menu in the `physics` program offers a sub-menu `Linear Regression`. Use length for the x variable, and period (time) for the y variable.

Three different analyses will be used for this data. If you select the `Perform regression and plot` option, period vs. lengh will be plotted with no modifications. The increase of period with increasing length is obvious, and one is tempted to let it go at that. However, you need to examine the parameters for the line: the slope and its standard deviation. Find the relative standard deviation (RSTD) of the slope. One should expect with our equipment that the RSTD would be on the order of 0.1% or even smaller. Something is seriously wrong with this analysis method.

Another analysis method will shed light on the problem. Using the `Transform data points` option, change the variables: for x, select `log(var)`, and for y, select `log(var)`. The display option now plots the base "e" logarithm of the period, log(period) vs. log(length). The RSTD of the slope should be pleasingly improved. This plot now tells us what sort of analysis we will need in order to extract a value for g from the experiment. Note the value of the slope: is it reasonably close to 1/2? If so, this suggests that the best graph would plot period vs. $(\text{length})^{1/2}$ or, equivalently, $(\text{period})^2$ vs. length. Print this plot for inclusion in your lab report.

Based on our result for the log(period) vs. log(length) plot, we now plot $(\text{period})^2$ vs. length. Select the `Transform data points` option, and change the variables: for the x variable, choose `original var`; then for the y variable, choose `(var)^n` where the power is 2. Now, you can plot $(\text{period})^2$ vs. length. Has the RSTD of the slope improved significantly? If so, record the slope and its standard deviation for this graph. Include this plot in your lab report.

This third option, $(\text{period})^2$ vs. length, is also suggested by the theoretical formula for the rod pendulum:

$$T^2 = \left(\frac{8\pi^2}{3g}\right) L \,, \tag{2.1}$$

where g is the acceleration of gravity, T is the period, and L is the length of the rod. If we accept this model for our rod pendulums, your slope can be set equal to the theoretical value, $8\pi^2/(3g)$ and you can solve for g.

Propagation of errors methods allow us to get an estimate for your value of g by using the amount of scatter of your data points from the ideal best line. Solve for the standard deviation for g:

$$\sigma_g = g\,(\text{RSTD of the slope})$$

where g is your value found above, and the RSTD of the slope is from your $(\text{period})^2$ vs. length plot. For example, with a value for g of $10.3245\,\text{m/s}^2$ and an RSTD of 0.002 (or 0.2%), you would express your answer as $g = 10.32 \pm 0.02\,\text{m/s}^2$.

Compare your result with literature value for g for our area. Does your result differ from the established value by more than one or two standard deviations? If so, what improvements should be explored? Is this method for finding g better than last week's system? Considering the vast improvement in the measurement system, is the error also much smaller? How much smaller?

There are editing tools available in the `Linear Regression` menu. For instance, if you notice that you have entered a point incorrectly, you can remove that point by selecting `Remove a data point`. By now, your screen is probably cluttered up with graphs. You can remove unwanted plots by placing the cursor over the plot and typing the letter `q`.

2.4 Propagation of Errors of Errors of Errors ...

The propagation of errors methods provide estimates of the effects of measurement errors on the results of an experiment. These methods will be introduced in the manual in greater detail as they are needed. Initially we give three general formulas:

1. When several items are combined by addition or subtraction, the standard deviation of the result is the square root of the sum of the squares of the individual standard deviations. For example:

$$Z = A + B - C \quad \text{and} \quad \sigma_Z = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_C^2}\;.$$

2. When several items are combined by multiplication and division, the relative standard deviation of the result is obtained by the same method as in 1 but using relative standard deviations. For example:

$$Z = \frac{AB}{C} \quad \text{and} \quad \frac{\sigma_Z}{Z} = \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_C}{C}\right)^2} \, .$$

3. A value raised to a power has a relative standard deviation for the result which is the relative standard deviation of the value times the power. For example:

$$Z = A^2 \quad \text{and} \quad \frac{\sigma_Z}{Z} = 2\frac{\sigma_A}{A} \, .$$

The `Math` menu of the `physics` program provides a propagation of errors calculator. Notice that ten different analysis types are offered: some involve combining two values (sum of two numbers), while some involve modifying a single value (square root of a value). After choosing one of the options, you are prompted by the program to supply the one or two values to be processed, along with their standard deviations. Your group should use the program to perform analyses on the following five exercises and include the results with your lab report:

1. $(4.6 \pm 0.2) + (2.3 \pm 0.1)$
2. square root of the above result
3. $(15.63 \pm 0.07) \times (5.61 \pm 0.02)$
4. this result $+ (52.6 \pm 0.3)$
5. $\dfrac{(1.2000 \pm 0.0002)^2}{(0.6000 \pm 0.0003)}$

Experiment 3

Static Forces

The objectives for this session are:

1. to define and measure the forces and torques acting in an equilibrium system,
2. to use the components of forces vectors to find total force, and
3. to interpret sources of error to show whether these equilibrium systems follow Newton's Law.

The simplest illustration of Newton's concept of force is in those systems in which the force vectors add up to zero, leaving no net force. In such systems there can be no acceleration, although there may be a (constant) velocity. In the case of zero velocity and acceleration, one observes a motionless system, suggesting the name "statics."

In this lab, we measure the forces acting on a motionless object. The analysis combines the separate forces as a vector sum to determine whether the forces in motionless objects do indeed cancel one another. Three types of experimental arrangements are provided to demonstrate different principles of statics. The three experiment stations can be studied in any order.

3.1 Forces Acting on a Point

The force table is a full-circle protractor table which permits pulleys to be attached anywhere on the edge. Weights suspended on strings over the pulleys apply a force to a small wire ring at the center of the table. The pulleys should be positioned so that the string going over each of them passes straight over the pulley. Four such forces are to be applied on the table, adjusting the weights and pulley locations until the ring returns to a position at the center of the circle when it is pulled to one side in any direction. Since the weights supplied with this apparatus are not very precise, measure the total weight on any string by removing the mass and weighing it on the electronic balance. In your lab notebook, prepare a data table with six columns:

which mass	M (g)	θ (degrees)	$\|\mathbf{F}\|$ (N)	F_x (N)	F_y (N)
1					
2					
⋮					

For each weight, enter the mass M in grams and the angle θ of the string. Make a sketch of the apparatus in your lab book, labeling the forces and angles.

Analysis

For the analysis, start by defining a coordinate system (x- and y-axes) in your diagram containing the force vectors. Next, you need to perform the following calculations:

1. For each weight, convert the mass to kg, then newtons, using g. This gives the magnitude of the force $|\mathbf{F}|$.
2. Multiply each force by the cosine of the angle for the x-component F_x.
3. Multiply each force by the sine of the angle for the y-component F_y.
4. Under each of the last two columns, compute the sum.

The sums for F_x and F_y should both be zero, to within errors.

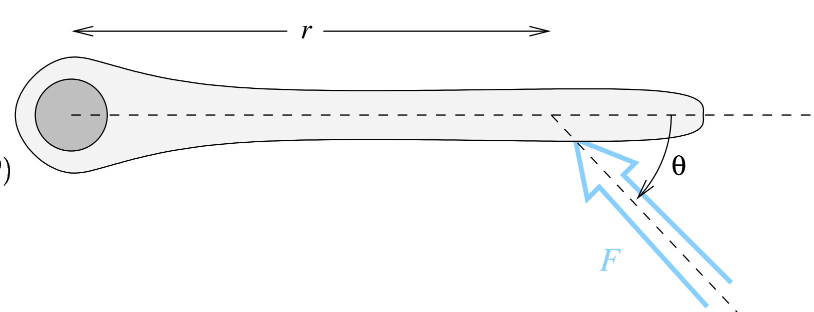
Several sources of error can be identified. The systematic error in the weights is eliminated if the weights are checked with the electronic balance. Assume that the error of the balance is ± 1 in the last digit of the reading. The angles are probably accurate to about one degree. Friction in the pulleys is a major error which is not easily reduced. Adjusting the weights to determine how much change shows up as a definite imbalance suggests that the equivalent of about 5 g of force, or 0.05 N of force may be introduced by the pulleys. This would suggest that the total force in the direction of either axis may appear to be as large as 0.05 N due to friction. The angular errors probably introduce smaller errors in the force than this.

Is the sum of the force components in each direction much larger than 0.05 N? What aspects of the experiment might cause discrepancies? Verify that friction is the major source of error by showing that it has the largest relative error of any effect mentioned above.

3.2 Torque Balancing

Another condition for equilibrium applies when the forces on an object do not all project back to a single point. The torque τ acting on an object is

$$\tau = rF\sin(\theta)$$



Forces pressing in the downward direction produce a negative torque while forces acting upward produce a positive torque. In equilibrium, all the torques must cancel one another.

A simple demonstration of this idea is the see-saw. Start by balancing a meter stick—without any weights— on a knife-edge to determine its center of mass. Record the position of the knife-edge on the meter stick. Next, arrange five different masses at various positions on the meter stick such that the system is still balanced at the same location of the knife-edge. In this case, all the angles between forces and their position vectors are $\pm 90^\circ$ and the sine of the angle is ± 1, making angle measurements unnecessary. Record the mass and the position (0–100 cm) along the meter stick for each of the five weights. Remember to check the weights using the electronic balance.

Analysis

Find the *displacement* of each mass relative to the position of the knife-edge. Weights to the left and right of the knife-edge have negative and positive displacements, respectively. Convert units to kilograms and meters. Multiply by g to find the force produced by each mass. Now calculate the torque due to each mass. Note that some torques are positive and some are negative; they should add up to zero, to within errors.

The errors are much smaller in this apparatus than for the force table because friction plays virtually no part. The weights used are generally reliable to better than 0.0001 kg. The angular contributions cancel exactly. The main source of error is in reading the position of each slotted weight relative to the knife-edge. For careful measurements in a system of weights in the range of 100 to 200 grams, none located too close to the knife-edge, distance errors would contribute approximately $0.005\,\mathrm{N}\cdot\mathrm{m}$ of error in the overall torque, an amount small in comparison with the individual torques being combined. How do your results compare with the $0.005\,\mathrm{N}\cdot\mathrm{m}$ limit suggested by this analysis?

3.3 Suspended Bar

In this system, both forces and torques contribute conditions for equilibrium, requiring both of the types of analysis used separately in the other parts of the experiment. Assemble the apparatus, making changes in the weights as desired to give a system which returns to equilibrium when perturbed gently in any direction. Check the the ropes move freely through the pulleys. You will need to weigh the masses on the scale.

Make a table for entering in rows each of the following: the mass of the weight M, the position relative to the left end of the bar r, and the angle of the force relative to the right horizontal θ.

which mass	M (g)	r (m)	θ (degrees)	$\lvert\mathbf{F}\rvert$ (N)	F_x (N)	F_y (N)	τ (N $\cdot$ m)
1							
2							
$\vdots$							

You must be careful with your protractor measurements. Forces to the left are between 90 and 180 degrees if they pull up or between 180 and 270 degrees if they pull down. Forces down to the right are in the range of 270 to 360 degrees, *et cetera*. Don't forget to include the bar itself as one of the masses; its weight is marked on the back. The position for its weight is taken as the black dot in its center, and its force is in the downward direction.

Analysis

Start your analysis by drawing a force diagram. Next, you need to perform the following calculations:

1. For each weight, convert the mass to kg, then newtons, using g. This gives the magnitude of the force $|\mathbf{F}|$.
2. Multiply each force by the cosine of the angle for the x-component F_x.
3. Multiply each force by the sine of the angle for the y-component F_y.
4. Multiply the y-component by the distance (m) from the left end of the bar for the torque τ.
5. Under each of the last three columns, compute the total F_x, F_y and τ.

The sums for F_x, F_y, and τ should all be zero, to within errors. The most difficult part of this experiment is in getting all of the signs correct.

Finally, we need to quantify the errors. The largest source of error is, again, friction due to the pulleys. This could be as large as 0.05 N. How does the sum of the forces compare to this error? How large do you expect the error for the sum of the torques to be?

3.4 Group Report

In discussing the sources of error in the experiment, avoid using the phrase "human error." That may work well in BIB112, but not here. To explain away large errors by citing difficulty in reading angles or distances flies in the face of tolerances that we have discussed above. You will have to be more creative in explaining discrepancies which are larger than two or three standard deviations.

Experiment 4

One Dimensional Motion on the Air Track

The objectives for this session are:

1. to learn to operate the air track system,
2. to measure small non-ideal effects in the system,
3. to define and measure velocity and acceleration,
4. to observe acceleration on an incline,
5. to interpret incline data to derive a value for g, and
6. to use error analysis to determine the quality of the measurements.

The air track is an instrument which permits the observation of motion with minimum interference from friction. It has two supporting planar surfaces, flat to within 0.002 in., with two sets of air holes spaced 1 in. apart. Air cars fit on the track with skirts which match the orientation of the track surfaces. Air supplied by compressors in another room is forced out through the holes in the track, lifting the car away from the track and supporting it on a cushion of air. The track and car are made of relatively soft aluminum metal; any distortions, gouges, or bent corners will cause the car to drag on the track. Because there is no direct contact between the car and the track, the car moves with very little friction and is extraordinarily sensitive to forces. This sensitivity permits the detection of several types of undesired forces. Small dips and humps in the track alignment are evident when the car is placed at different locations on the surface: the car may drift slowly to the left at one position, then to the right at another. This has no detectable effect on results of experiments which cover the whole length of the track, but these local imperfections make it difficult to adjust the track to be perfectly level such that the right end of the track is exactly at the same height as the left. The residual overall tilt of the track has to be measured by special experiments. Although friction is reduced in this system, it is still present due to air film viscosity. The car experiences a small force as it moves, in proportion to its speed, because it distorts the thin layer of air on which it is supported. With precision time and velocity measurements it is possible to measure both the effect of the air film friction and any small residual tilt in the track.

Because the air track is the ideal apparatus for demonstrating many physical systems, we will use it for several experiments and general directions are given here for the use of the track:

1. After the air supply is turned on, use a slightly dampened paper towel to remove dust from the supporting surfaces, avoiding forcing any grit into the air holes.

2. Select a car and place it gently on the track. If the car tends to drift in one direction no matter where it is placed on the track, adjust the level of the track. With the track approximately leveled, set the car in slow motion. Watch for any locations at which the car slows suddenly as it moves over the entire length of the track. If the car is to carry extra weight, perform a test run with the car carrying its maximum added weight. If the car slows at any spots on the track, try turning the car around; it often works better in one orientation than in the other. If it still does not move smoothly, examine the car for damaged corners and the track for gouges. Select another car, or have the instructor repair any track surface damage.

3. Avoid damage to the car and the track. Place the car gently on the track or on the table when not in use; do not drop the car, especially on the track.

4. When directions call for a preliminary leveling of the track, place the motionless car, with no attachments, at several locations on the track, observing its tendency to drift left vs. right. The track is approximately level when the tendencies for drifting to the right and left cancel at several sampling locations over the whole length of the track.

5. Avoid speeds in excess of 1 m/s, beyond which collisions at the end of the track may cause damage to the car, the bumper springs, or the track.

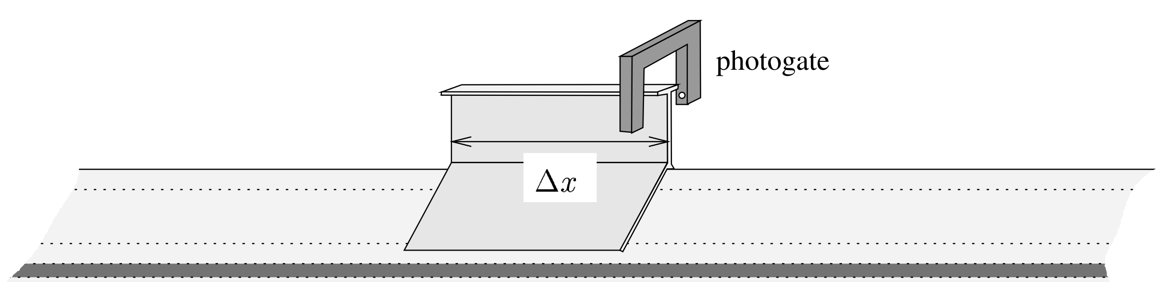
6. Note that the ruler on the airtrack has three sections, each one meter long: you must add 0, 1, or 2 m depending on the part of the rule that you are reading.

4.1 Constant velocity motion on the level track

Velocity will be measured by determining the amount of time that the car is blocking the photogate beam. If the effective length of the car is Δx and the time required for the car to pass through the beam is Δt, then the average velocity during that time interval is

$$v = \frac{\Delta x}{\Delta t} \, .$$

In this part of the experiment, two photogates are mounted on the track near the two ends. The computer notes the time, on its internal clock, at which the car blocks a beam or unblocks a beam.



In order to study the differences in motion in the two directions, two velocities will be obtained as the car moves from left to right, then two more velocities will be obtained as the car returns in the opposite direction. The time intervals observed will all be nearly equal, indicating nearly constant velocity, but small differences can be analyzed to detect any residual tilt in the track and to measure the small effect of air film friction.

We will assume the sign convention that motion to the right is taken as positive velocity and, if the speed is increasing to the right, the acceleration is positive. Let a_L and a_R be the acceleration while the car is moving to the left and right, respectively. The average accleration is defined to be:

$$\text{acceleration} = \frac{\text{velocity at first photogate} - \text{velocity at second photogate}}{\text{time at first photogate} - \text{time at second photogate}} . \tag{4.1}$$

Friction is always opposed to the velocity: if the motion is to the right, friction gives *negative* acceleration, while if the motion is to the left, friction gives *positive* acceleration. Consequently, $(a_L - a_R)/2$ gives the magnitude of the friction acceleration. If the track is higher on the left end, the resulting acceleration is positive no matter which way the car moves: $(a_L + a_R)/2$ gives the average acceleration due to any tilt of the track.

Procedure

1. Prepare and check the track and car as described above in the general directions; adjust the track to make it as level as possible, using a car without bumper springs.

2. Mount the photogates at positions 30 to 40 cm from the two ends of the track. Adjust the heights of the photogate beams such that the car blocks the beam at a point 0.5 inch below the top of the car.

3. At the left photogate, move the car to the right slowly until the car just interrupts the beam, indicated by the red lamp on the beam housing. Read the position of the right corner of the car, in meters, on the metric rule on the track. Move the car on through the beam slowly until the red lamp goes off, and take a second reading; the difference of these two readings is Δx, the effective length of the car. Record Δx and estimate its error.

4. Practice launching the car, allowing it to continue bouncing from the ends, noting that it moves gradually more slowly after each collision with an end. A good speed for these observations is such that it takes 4 to 6 seconds to cover the length of the track.

5. Run the program `physics`, select `Air track experiments`, and `Level the air track.` Follow the instructions as to when to press the `Enter` key to begin time measurements. The program notes the times at which the car blocks the first beam, the second beam, and the first beam while returning in the opposite direction. The data are then analyzed to determine the acceleration while the car is moving in each of the two directions. These accelerations provide estimates for two very small effects: friction of the air film causes a diminishing of the speed in both directions, while residual tilt causes increasing speed in one direction and decreasing speed in the other direction.

6. Repeat the procedure for five runs, noting the results for friction (beta), tilt acceleration, and estimated tilt for each run. Also, record all of the the times and velocities for *one* run.

Analysis

The computer program calculates the friction parameter β and residual tilt y in the following manner. Comparing accelerations in the two directions leads to the friction parameter (fraction of speed lost each second)

$$\beta = -\frac{a_R - a_L}{v_R - v_L} , \tag{4.2}$$

where v_R is the average velocity as the car moves to the right and v_L is the average velocity as the car moves to the left. Remember that velocities are signed: we take a velocity to the right to be positive. The acceleration due to tilt is found in a similar manner:

$$a_{\text{tilt}} \approx \frac{a_L + a_R}{2} . \tag{4.3}$$

We can use this to estimate how much higher one side of the track is:

$$y = \frac{a_{\text{tilt}}}{g} \cdot 90\,\text{in.} , \tag{4.4}$$

where 90 in. is the distance between supports and g is the acceleration of gravity.

From your results for β and y, compute the resulting average values and standard deviations. The standard deviation will serve as an estimate of the error in measuring β and y.

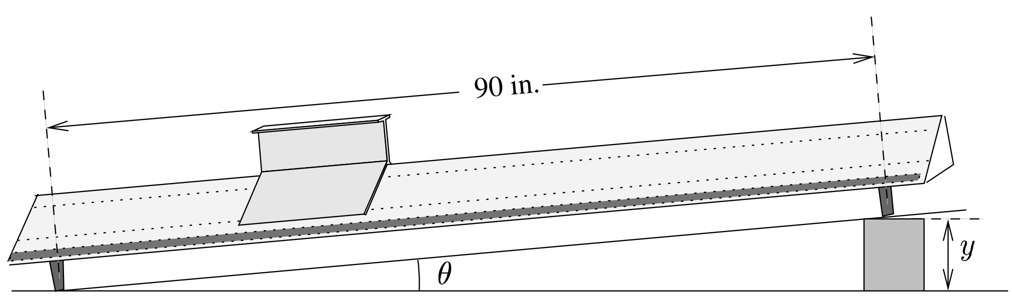
In the analysis section of your report, take *one* run and calculate β, a_{tilt}, and y using Equations (4.2), (4.3), and (4.4) above. Does your answer agree with the computer program?

4.2 Constant Acceleration on the Inclined Track

When the air track is inclined by lifting one end, motion with constant acceleration should be observed:

$$a_{\text{tilt}} = g \sin(\theta) , \tag{4.5}$$

where θ is the change in angle obtained by putting riser blocks of height y under the end.



Using trigonometry,

$$\sin(\theta) = \frac{y}{90\,\text{in.}} , \tag{4.6}$$

where y is expressed in inches. When the acceleration of the car is observed on the inclined track it is possible to obtain a fairly accurate value for g from this motion.

Procedure

1. Do not make any shifts in the position of the track from the first part of the experiment. Place two 1.000 in. aluminum riser blocks under the single foot end, moving the other end as little as possible. Because some of he recent riser block acquisitions have been irregular, you need to check the thicknesses with calipers: measure in cm, then convert to inches using the identity $1\,\text{in} = 2.54\,\text{cm}$.

2. Note whether the left or right end of the track is tilted. This will affect your analysis.

3. Start with the car at the bottom of the hill and give it a *gentle* push so that it has just enough energy to move past the upper photogate. It is essential that you avoid the unpleasant sound of the car scraping against the track. Ouch!

4. In the `physics` program, choose the menu item `Velocity studies on the airtrack`. Give the car a gentle push so that it has just enough energy to go through both photogates and return to its starting position. Record the various times in your lab notebook.

5. To stop recording times, just press the `Enter` key.

6. Repeat Step 4 five times, using the same starting position, checking the results for consistency. The mean and standard deviation of the acceleration for several runs will be calculated in the analysis.

Analysis

We now have another way to measure g. Start by calculting the velocities, expressed in meters/second. The velocities at photogates are measured as the car length divided by the time intervals for which the respective beam is cut. Remember that velocities to the left are negative. Use this to calculate the average acceleration, Eqn. (4.1), for both the uphill and downhill motion, obtaining a_R and a_L for each run. Your results for a_R and a_L should be quite similar.

Recall that friction will increase the uphill acceleration and decrease the downhill acceleration. Thus, the average $a_{\text{tilt}} = (a_R + a_L)/2$ should be a good estimate of the acceleration due to gravity since the frictional effects will cancel. Find a_{tilt} for each run and calculate the average and standard deviation.

Next, combine Equations (4.5) and (4.6), and solve for g using your average value of the acceleration. If we take the relative standard deviation for g to be the same as the relative standard deviation for a,

$$\frac{\sigma_g}{g} = \frac{\sigma_a}{a} ,$$

we can solve for σ_g, the standard deviation of g, the only unknown variable in this equation. Report the value for g and its standard deviation σ_g.

4.3 Group Report

The data section of your report should include complete details for *one* run of each of the two experiments. For these two runs, the analysis section should include all calculations of velocities, accelerations, *et cetera*. For the remaining runs, you only need to list the final results:

- For the first experiment, you should include values for β, a_{tilt}, and estimated tilt. Calculate the mean and standard deviations for these quantities in the analysis section of your report.

- For the second experiment, include a_L and a_R for each run.

The discussion (conclusion) should include comments on the quality of the measurements, how well they compare with previous experiments, and suggestions for refinement or variation of the experiment.

In Section 4.1, two very small effects are sought. Do the repeated measurements indicate that the results are real? If the mean is not at least twice its accompanying standard deviation, the mean is virtually zero, and the effect is not measurable. The residual tilt depends upon how well the track was originally leveled and may well be undetectable.[1] Based on your results for residual tilt and for β, comment on whether these effects are measurable.

In Section 4.2, you have yet another approach to measuring g. Considering the reproducibility for the measured value of g observed in repeated runs, what do you conclude about the quality of this method for measuring g compared to the two previous experiments? Are there defects in our model for the system, or are the measurement procedures inherently better/worse than those used in Experiment 2? In describing the quality of the results, we are more interested in how small is the relative standard deviation for g, rather than how well its value agrees with the literature. If the standard deviation is small but the value is wrong, then there is some calibration (systematic) error that undermines the accuracy of the result. A typical source of calibration error would be uncertainty in the thickness of the riser blocks. Can you think of other sources?

[1] That is, the measurement error is larger than the value.

Experiment 5

Conservation of Momentum

The objectives for this session are:

1. to measure quantities used to define the momentum and kinetic energy of objects,
2. to determine whether momentum is conserved during collisions, and
3. to determine whether kinetic energy is conserved during collisions.

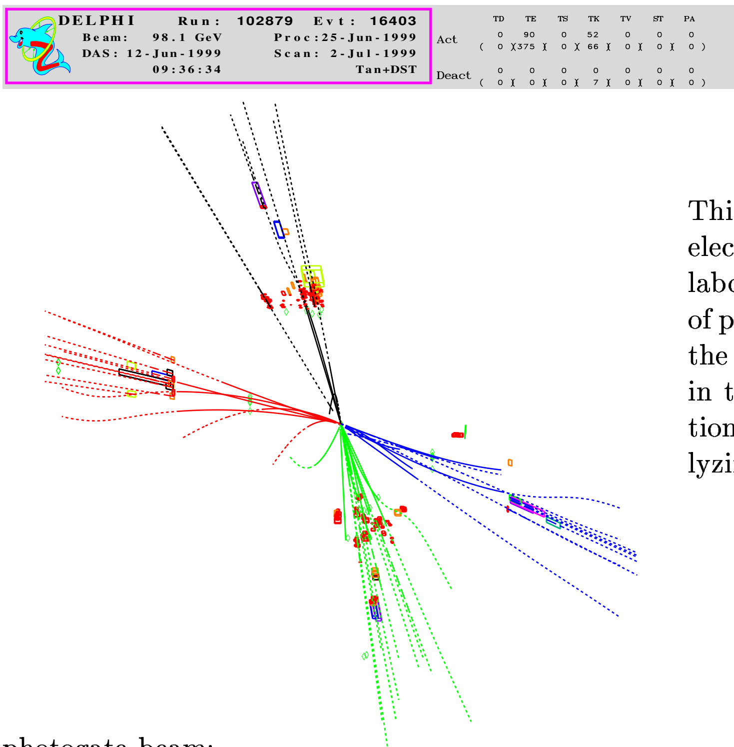
5.1 Inelastic and Elastic Collisions

Two widely separated objects move toward each other, they collide, and something emerges from the process. Collisions arise in a wide variety of situations: galaxies colliding in space, particles colliding in an accelerator, molecules hitting each other in a gas, or—in today's lab—cars hitting each other on an airtrack. In this lab, we shall discover that collisions can be understood in terms of two quantities: energy and momentum.

In the absence of external forces, the total momentum of any system does not change during a collision. An object with mass m moving at a velocity $\mathbf{v}$ has momentum $\mathbf{p} = m\mathbf{v}$. Note that momentum—like velocity—is a vector quantity. For one-dimensional motion, motion to the right can be taken as positive momentum, while motion to the left is negative momentum. The total momentum of several objects is found by adding the momentum of each interacting object. In this lab, you will measure the total momentum before and after a collision. If momentum is conserved, the two measurements of the momentum will be the same.

Another important property of moving objects is the kinetic energy. The kinetic energy of an object is $T = \frac{1}{2}m\mathbf{v}^2$. The total kinetic energy is the sum of the energy of each object. Since kinetic energy is a positive, scalar quantity, you won't have to worry about minus signs. If the total kinetic energy of the system is found to be the same before and after a collision, the collision is called *elastic*; otherwise it is said to be *inelastic*. For inelastic collisions, the missing kinetic energy is converted into something else: maybe heat, new particles, or rotational energy. Although the total energy is always conserved, kinetic energy is not.

In this study of momentum and kinetic energy, several quantities need to be measured. For each object, before and after the collision, the mass and velocity must be known. The velocity of a car is obtained by measuring the time interval Δt during which the car, length Δx, is in the



This figure shows particles produced by an electron-positron collision at the CERN laboratory in Switzerland. The four "jets" of particles emerging from the collision are the decay products of four quarks created in the collision. Physicists use conservation of energy and momentum when analyzing these collisions.

photogate beam:

$$v = \pm \frac{\Delta x}{\Delta t}$$

where motion to the right corresponds to positive velocity. You will use the `physics` program, menu `Air track experiments`, sub-menu `Velocity studies on the airtrack` to measure Δt. If an object is motionless, its velocity is assumed to be zero without measurement.

5.2 Procedure

Prepare a table in your notebook with at least 20 rows:

car	Before				After			
	Δt (s)	velocity (m/s)	momentum (kg · m/s)	kinetic energy (J)	Δt (s)	velocity (m/s)	momentum (kg · m/s)	kinetic energy (J)
car A	⋯	⋯	⋯					
car B	⋯	⋯	⋯					
total			⋯	⋯			⋯	⋯
⋮								

5.2.1 One car at rest

1. Choose two cars (labeled "A" and "B" in the table) that are the same size, both fitted with regular springs.

2. Check the level of the track.

3. Weigh the cars and record the masses, in kilograms.

4. Measure the lengths of the two cars by moving them through a photogate and noting the position as the photogate LED turns on and off.

5. Place one car *at rest* in the region between the photogates, $\mathbf{v} = 0$.

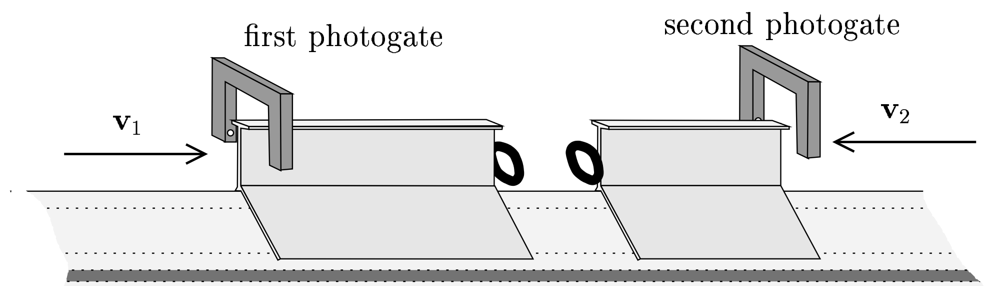
6. Launch the other car so that it passes through one of the photogates and collides with the first car. Be sure that neither car is blocking a photogate *during* the collision.

7. Record the velocity of each car before and after the collision in your table.

8. Repeat until you have at least four good runs.

5.2.2 Collision without coupling

1. Select two different sized cars with regular springs on each car.



2. Weigh the cars.

3. Measure the lengths of the two cars.

4. Launch both cars so that they pass through the photogates and collide in the region between the photogates.

5. Record the initial and final velocities of the cars.

6. Repeat until you have at least four good runs.

5.2.3 Coupled Cars

1. Select two different sized cars and attach springs with sticky-stuff to the ends of the cars.

2. Measure and record the masses of the two cars in kilograms.

3. Measure the length of both cars by moving them through the photogates.

4. As before, gently launch the cars so that they pass through the photogates and collide in the region between the photogates. Perform a couple practice runs to find the best speed for the collision. Excessive speed will cause the cars to come apart during the collision. An initial speed which is too small will magnify the effects of friction and any imperfections of the track.

5. Record the velocity of each car before and after the collision in your table. In this case, the velocity of the two cars afterwords will be the same.

6. Repeat until you have at least four good runs.

5.3 Analysis

For each run, calculate the initial and final momentum and kinetic energy for each car. Then find the *total* momentum before and after the collision. For each run, compare the total initial momentum with the total final momentum. Finally, for each run, compare the total initial kinetic energy with the total final kinetic energy.

Take *one* of your runs and calculate the relative error in your measurement of momentum and velocity. Let us assume that the measurement of mass has error 0.5 g, car length Δx has error 0.1 cm, and the time Δt has error 0.001 s. The error in the velocity σ_v is given by the formula:

$$\frac{\sigma_v}{v} = \sqrt{\left(\frac{0.0005\,\mathrm{s}}{\Delta t}\right)^2 + \left(\frac{0.1\,\mathrm{cm}}{\Delta x}\right)^2}\,.$$

Make sure you get your units right! The error in momentum p is found using

$$\frac{\sigma_p}{p} = \sqrt{\left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{0.5\,\mathrm{g}}{\mathrm{mass}}\right)^2}$$

and the error in kinetic energy is found using

$$\frac{\sigma_T}{T} = \sqrt{4\left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{0.5\,\mathrm{g}}{\mathrm{mass}}\right)^2}\,.$$

Since the velocities and masses in the runs roughly similar, you may use these as rough error estimates for *all* of the runs in both parts of the experiment.

5.4 Group Report

The Data section of your group report should contain a table giving the initial and final momenta and kinetic energies for each run. You do not have to include the "raw data" that you collected.

In the Analysis section, show your calculation of the error for momentum and energy. Also, include an example from one run of how you calculated momentum and kinetic energy.

For each experiment, discuss whether momentum or kinetic energy seemed to be conserved. Be quantitative! Note that the test for conservation is that the change of total momentum (or total kinetic energy) is within 2 or 3 standard deviations of zero.

In your conclusion, use conservation of momentum and kinetic energy to explain what happened in Section 5.2.1, assuming the two cars had identical masses. Why did the cars behave as they did?

Experiment 6

Energy Exchange in Mechanical Systems

The objectives for this session are:

1. to collect data for calculating energy in different forms,
2. to examine systems which transform energy to determine whether energy is conserved, and
3. to study simple springs as energy reservoirs.

6.1 Introduction

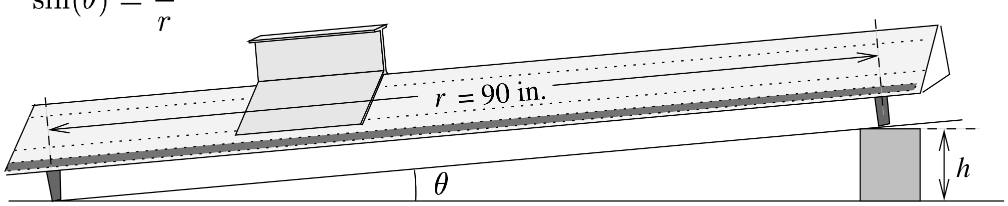
We will study three types of energy in this experiment: gravitational potential energy, spring potential energy, and kinetic energy.

Gravitational potential energy U_g is the work done in exerting a force against gravity while lifting an object. For an object of mass m moving a vertical distance h against the force of gravity, $U_g = mgh$. If an object is moved a distance S up a frictionless incline against the force of gravity,

$$U_g = mg\,S\,\sin(\theta)\ ,$$

where θ is the angle of incline above the horizontal.

$$\sin(\theta) = \frac{h}{r}$$



Kinetic energy T is the work done in setting an object into motion. It is the energy that an object with mass m has by virtue of being in motion at velocity $\mathbf{v}$:

$$T = \frac{1}{2} m \mathbf{v}^2\ .$$

Potential energy of a spring U_s is the work done in compressing a spring. For an ideal spring, the force F required to compress the spring a distance $x - x_0$ is

$$F(x) = k(x - x_0)\ ,$$

where k is the spring constant. (Here, F is the force acting on the spring; that is why there is no minus sign.) The work done in compressing the spring is

$$U_s = \int_{x_0}^{x} F(x')\,dx' = \int_{x_0}^{x} k(x' - x_0)\,dx' = \frac{1}{2}k(x - x_0)^2 \ .$$

In our experiment, the change of length of the spring will be monitored by how far the car compresses the spring. Thus, x is the position of the car (according to the track rule) and x_0 is the position of the car at the beginning of the compression process.

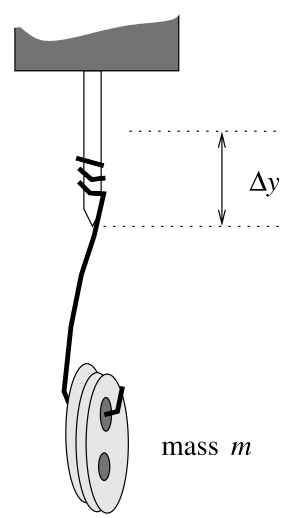
In this experiment, energy will initially be stored in the spring by pressing the car against the spring; the energy of the system at this point is U_s. When the car is released, the spring sets the car into motion: the energy that was initially in the spring now becomes the kinetic energy of the car T. Another variation is to perform the same experiment on an inclined track. Now the energy of the spring becomes kinetic energy in the moving car, but the car gradually slows as it moves up the incline, converting its kinetic energy into gravitational potential energy U_g. If energy is conserved, the total energy should be the same whether it is in the spring, in the moving car after it is launched, or in the gravitational potential energy of motionless car at the high end of the track.

In the first part of the experiment, we will calibrate the spring mechanism. We will suspend the spring mechanism vertically and add weights, mass m, to it. Due to gravity, the weights will exert a force $F_g = mg$ on the springs. This will cause the springs to stretch by the amount Δy. Using Hooke's law for springs,

$$F_g = k\,\Delta y \ , \tag{6.1}$$

we can determine the spring constant k.

6.2 Calibration of the spring mechanism

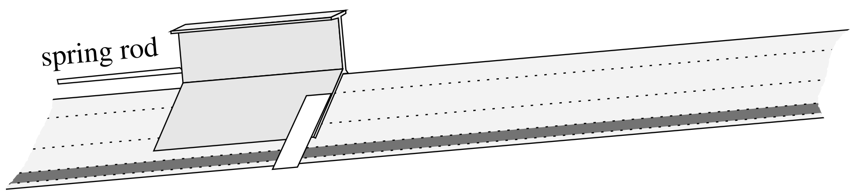


1. Clamp the spring mechanism to one of the rod clamps.
2. Attach a wire hook to the end of the glass rod so that weights can be hung from the glass rod.
3. Use a meter stick and *carefully* measure the position of the glass rod.
4. Add some weights to the wire hook, making sure that none of the springs is completely compressed.
5. Measure the position of the end of the glass rod again. The change in position is Δy
6. Use Equation (6.1) to determine the spring constant k.

Since the rest of the lab depends on getting this measurement right, perform this measurement twice, with different lab members doing the measurement the second time. Make sure that you got your units for k right!

6.3 Conversion of spring energy to gravitational energy

1. Clean the track and level it using a small car.
2. Place the spring mechanism on the track at the end with two feet. Fasten the mechanism in place with tape. Be sure that no tape gets on the track surface itself.
3. Place a 1 in. machined aluminum block under the single foot end. Record as m the mass of the car.
4. Place a small marker on the edge of the car so that you can read its position on the rule more accurately.



5. Determine x_0, the position where the car just starts pressing against the glass rod in the spring mechanism.
6. Practice launching the car and following its trajectory. For example, press the car back about 5 cm from x_0 and use a clean release to launch it. The car will climb the incline, coming to rest briefly, then move down the incline again. Catch the car before it hits the plunger. Note that, while the initial launch position can be read accurately, the position at the top of the trajectory is more difficult to read because the car does not stay at that position. In the analysis it turns out that while the initial position has to be read very accurately, the position at the top of the trajectory x_f needs to be read only to within a few millimeters. If you are able to launch the car repeatedly from the same position with a clean release, x_f will be fairly reproducible.
7. Prepare a table with four columns and eight rows.

starting position x_i (m)	final position x_f (m)	spring energy (J)	gravitational energy (J)
⋮	⋮		

8. Perform the measurements for several different compressions. For example, you could try $x_i - x_0$ equal to 3 cm, 5 cm, and 7 cm. Repeat each measurement to verify your results.

Analysis

For the inclined track experiment, the initial potential energy of the spring should match the final gravitational potential energy. Theory suggests that compressing the springs twice as much should make the car go four times as far up the incline. Do your results confirm this?

6.4 Conversion of spring energy to kinetic energy

1. Remove the riser blocks. Check the level of the track again.

2. Mount a single photogate at a position such that the car passes through the beam soon after the car uncouples itself from the spring mechanism.

3. Determine x_0 again.

4. Press the car against the spring plunger, observing the springs inside the box. Note how far you can move the car from x_0 before the spring windings begin to touch one another: all launchings should be smaller than this compression displacement.

5. Use a compression which is a little smaller than this maximum value. For example, if the maximum compression displacement was found to be 8 cm, use about 6 cm. Now with the car pressed about 6 cm into the springs from x_0, read and record as accurately as possible the starting position x_i.

6. Practice releasing the car a few times. A clean launch is required, such that very little of the spring energy is lost to your hand. One way to do this is to place a ruler or other hard object on the track to hold the car as it sits at its starting position. Now pull the ruler away in the direction that the car will be moving.

7. Use the `physics` program, menu `Air track experiments`, item `Velocity studies on the airtrack` to measure the velocity of the car. Try several runs for the same starting point—recording the velocity for each—to verify that your results are reproducible.

8. Repeat step 7 using a compression which is about half as large. For example, if 6 cm was used above, try 3 cm. Record the exact starting position x_i, along with the time intervals for several runs.

Analysis

For the level track experiment, the conservation of energy suggests that the initial spring energy should be the same as the final kinetic energy. How well do the two agree?

One source of error in today's experiments is the fact that the springs themselves have some mass. Thus, when accelerating the car, the springs themselves gain some kinetic energy. For example if the springs have a mass of 15 g and the car has a mass of 180 g, the fact that the spring system is moving after the car leaves suggests that only the fraction $180\,\text{g}/(180\,\text{g} + 15\,\text{g})$ of the spring energy is transferred to the car, about 93%. Would this be a source of *systematic* or *statistical* error?

6.5 Group Report

The Data section of your report should include the positions, x_0, x_i, x_f, *et cetera*, the velocities (for the level track part), and values for the energies. It should also include the raw data for your determination of the spring constant.

The Analysis section of your report should include your determination of k and an example of how you determined the energy for each part of the experiment. You might want to write out this part of the report by hand.

In the discussion section, compare initial and final energies for the various runs and comment on which parts seem to work best. Discuss any differences in your initial and final energies: are they larger than the estimated error? Don't forget to discuss the various questions that are asked in the Analysis sections.

Experiment 7

Rotational Dynamics

The objectives for this session are:

1. to measure the rotation of objects,
2. to calculate the moment of inertia on an object, and
3. to measure experimentally the moment of inertia of an object.

7.1 Measuring the Moment of Inertia

In order to describe an object rotating about a fixed axis, we define its angle of rotation $\theta(t)$, the angular velocity $\omega(t)$, and angular acceleration $\alpha(t)$, where

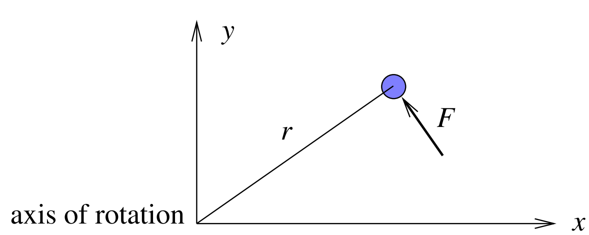
$$\omega(t) = \frac{d}{dt}\,\theta(t) \quad \text{and} \quad \alpha(t) = \frac{d}{dt}\,\omega(t)\ .$$

These variables are just like the usual position, velocity, and acceleration vectors that we have been using so far in this class, except that they are for rotation: $\theta(t)$ has units of radians, $\omega(t)$ has units radians/s, and $\alpha(t)$ has units radians/s^2.

As you know, we use Newton's second law, $\mathbf{F} = m\mathbf{a}$, to describe how objects move. There is an analogous law for rotational motion:

$$\tau = I\alpha$$

where: τ is the "torque" applied to the object. Torque is obtained by applying a force F to an object at a distance r from its axis of rotation, $\tau = Fr$.



The moment of inertia I is the rotational "mass" of an object.

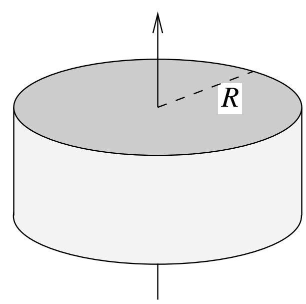
One objective of this lab is to find the moment of inertia I for various objects. The moment of inertia of n particles is given by the expression

$$I = \sum_{i=1}^{n} m_i r_i^2 \; , \tag{7.1}$$

where m_i is the mass of particle i and r_i is the distance of particle i from the axis of rotation. You can see from Equation (7.1) that I has units of $\mathrm{kg \cdot m^2}$/ Using this formula, one can, in principle, calculate the moment of inertia of any object. Some useful examples are given below:

- Disk or solid cylinder, mass M, and radius R:

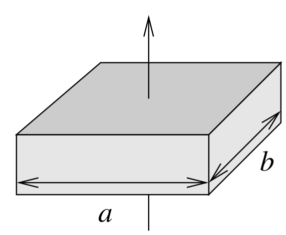
$$I = \frac{1}{2} M R^2 \; .$$



- Rectangular block, mass M:

$$I = \frac{M}{12}\left(a^2 + b^2\right)$$

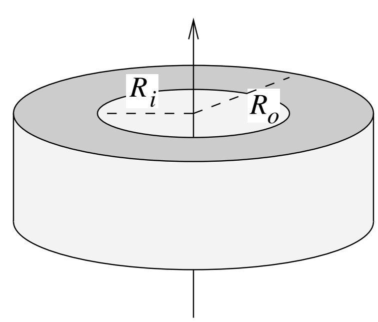
where a and b are the lengths of the edges.



- Disk, mass M, with a hole in the center:

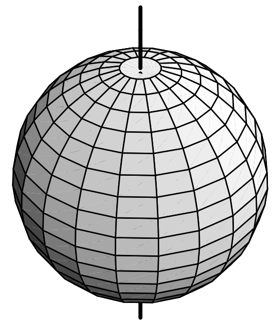
$$I = \frac{M}{2}\left(R_o^2 + R_i^2\right)$$

where R_o and R_i are the outer and inner radii.



- Sphere with mass M and radius R:

$$I = \frac{2}{5} M R^2 \ .$$



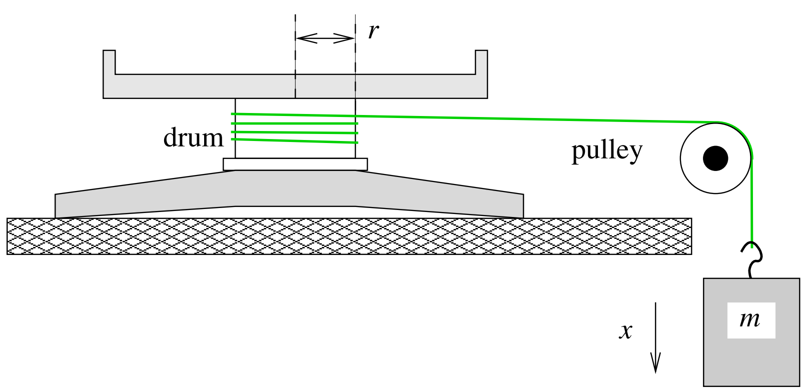
In each case, we assume the object has uniform density throughout. For irregularly shaped objects or those with nonuniform density, such formulas are difficult to generate; instead one can measure the moment of inertia experimentally.

In general, to measure the mass of an object, we see how much force is needed to accelerate an object a given amount. For instance, in a laboratory scale, the acceleration g is provided by gravity (using Einstein's picture of gravity) and we find the normal force F that is provided by the scale. The mass is given by $m = F/g$. For the moment of inertia, we do something similar, we see what torque τ is needed to produce an angular acceleration α. The moment of inertia is given by $I = \tau/\alpha$.

Two experimental arrangements will be used to demonstrate this method. The first is the rotational turntable in which a hanging weight applies a torque to a rotating sample, causing it to move with a measurable angular acceleration. The second is the torsion pendulum where a sample is suspended from an elastic wire. The object undergoes rotational oscillations and one measures the period of oscillations. The moment of inertia obtained from these experiments will be be compared with the geometric formulas listed above.

7.2 Rotational Turntable

You will use the program `physics` program, menu item `Rotation experiments`, to measure the motion of the turntable. In this experiment, several flags mounted on the turntable interrupt the photogate. This is used to measure the angular velocity of the turntable.



Procedure

1. Use calipers to measure the diameter of the turntable drum. Divide by two to obtain the *radius* of the drum r.

2. Position the photogate assembly to permit the turntable flags to cut the beam without interfering with the motion.

3. The first step is to measure the moment of inertia of the empty turntable. You will have to experiment with the amount of weight to use, try to find an amount so that it takes about two seconds for the three rotations. Record your results.

4. The computer program displays the times that the beam is blocked and the amount of time that it is blocked. You can use the initial and final angular velocity ω to find the angular acceleration α. For uniform angular acceleration (which is what we have here),

$$\alpha = \frac{\omega(t_1) - \omega(t_0)}{t_1 - t_0} \; .$$

5. Perform a few runs to make sure your value for α is reproducible.

6. Next, choose a big ring or disk to place on the turntable. Measure the mass of your sample and its radius, or its inner and outer radii if it is a ring.

7. Now measure α for your sample object. Adjust the falling mass so that that it takes 2–4 seconds for the three rotations. Perform multiple runs to determine the reproducibility of the results.

8. Try some runs with a very different value for the falling mass.

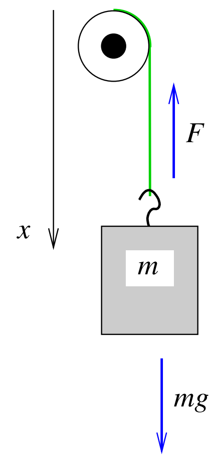
Analysis

If we measure the angular velocity ω at times t_0 and t_1, we can find the angular acceleration,

$$\alpha = \frac{\omega(t_1) - \omega(t_0)}{t_1 - t_0} \; .$$

This, multiplied by the drum radius r, gives the acceleration of the falling weight, $a = r\alpha$.

The falling weight, mass m, has two forces acting on it: the tension F of the string pulling upwards and the force of gravity mg pulling downwards. Using Newton's second law (with the coordinate x pointing downwards), we have



$$mg - F = ma$$

or

$$F = m(g - a) \ .$$

The force F gives rise to torque on the turntable of

$$\tau = Fr = m(g - a)r = m\left(g - r\alpha\right) r \ ,$$

where r is the drum radius. Using this, we can find the moment of inertia,

$$I = \frac{\tau}{\alpha} \ .$$

1. For each of the runs, use the formulas to find τ and α.
2. For the empty turntable, divide the torque τ by α to obtain the moment of inertia for the empty turntable I_t.
3. For the turntable with sample, divide τ by α. This gives $I_t + I_s$, the sum of the moments of inertia of the turntable and the sample.
4. Subtracting the result of Step 2 from the result of Step 3 gives the moment of inertia of the sample I_s.

Compare your results for I_s with the geometric estimate using the formulas in Section 7.1. What is the percentage difference between these values? What sources of error can you suggest to explain the difference, and what improvements in procedure would give better agreement?

You used both small and large falling masses to measure α. Which measurement gave you better estimates of the moment of inertia?

7.3 Torsion Pendulum

You can use the `physics` program, menu item `Pendulum experiments`, to measure the period of the torsion pendulum.

1. Practice setting the holder into motion and measuring the period with the photogate timer; wobbles are bad. Be sure that the flag blocks the beam when the pendulum is motionless, and that the photogate does not interfere with the motion. Use small amplitudes of about 1 in.
2. Find the period of oscillations for the empty holder. We will call this period T_e.
3. Measure the mass and inner and outer radii of the standard ring sample. Use the formulas in Section 7.1 to determine its moment of inertia I_r.

4. Find the period of the pendulum with the standard ring. We will call this T_r. Be sure that sample is centered on the holder.

5. Measure and record the mass and dimensions (radius, radii, or length and width, depending on the shape of the of the object) of one unknown sample.

6. Find the period of the pendulum for this sample. We will call this T_s.

In each case perform a couple of runs to ensure your results are reproducible.

Analysis

The formula for the period of the torsional pendulum is similar to that of the ordinary pendulum:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

where I is the moment of inertia of the holder and its sample, in $\mathrm{kg}\cdot\mathrm{m}^2$, and κ is the torsion constant of the wire in $\frac{\mathrm{N\cdot m}}{\mathrm{radian}}$. We can square this expression to obtain:

$$I = \frac{\kappa}{4\pi^2}T^2\ ;$$

we will use this formula below.

The apparatus has two unknown parameters which must be fixed by calibration before moment of inertia of any sample can be determined: the moment of inertia of the empty holder I_e and the torsion constant κ. In Step 2, we found the period of the empty holder. Thus,

$$I_e = \frac{\kappa}{4\pi^2}T_e^2\ . \tag{7.2}$$

In Step 3, we found the period for the standard ring; the associated moment of inertia is $I_r + I_e$. Thus

$$I_r + I_e = \frac{\kappa}{4\pi^2}T_r^2\ . \tag{7.3}$$

Subtracting Equation (7.2) from Equation (7.3), we obtain

$$I_r = \frac{\kappa}{4\pi^2}\left(T_r^2 - T_e^2\right)$$

so

$$\kappa = \frac{4\pi^2 I_r}{T_r^2 - T_e^2}\ . \tag{7.4}$$

In the same manner, the moment of inertia of any unknown sample is given by,

$$I_s = \frac{\kappa}{4\pi^2}\left(T_s^2 - T_e^2\right)$$

where the value of κ given by Equation (7.4). Use this expression to find the moment of inertia of your unknown sample.

Now, use the formulas in Section 7.1 to find the moment of inertia of your unknown sample. Find the percentage differences between your measurement using the torsion pendulum and the geometric estimate.

7.4 Group Report

For your group report, summarize moment of inertias obtained from the two methods. How reproducible were your results? How well did they compare with the moment of inertia obtained using the formulas in Section 7.1. Does the torsion pendulum give better agreement than the turntable method? What features of the two measurement systems would you cite to explain why one system gives better results than the other?

Experiment 8

The Harmonic Oscillator

The objectives for this session are:

1. to find how spring constant, mass, and amplitude affect the period, and
2. to determine the effect of friction on the amplitude.

8.1 Period and Velocity for a Spring Oscillator

Many things in the world around us can be understood in terms of oscillations: the motion of waves, the vibrations of molecules, the orbits of stars and galaxies, the behavior of electric circuits, *et cetera*. We have already seen several examples in previous experiments: the ball and string pendulum in Section 1.4, the rod pendulum, and the torsion pendulum. In this experiment, we will study the motion of a car on the airtrack with springs attached to each end. For sufficiently small oscillations, springs of any kind produce a force of the form

$$F = -k\,\Delta x$$

where F is the force produced by the spring, Δx is the displacement from equilibrium, and k is the spring constant. The minus sign indicates that F is in the opposite direction of the displacement Δx; the spring tries to contract when stretched and tries to expand when compressed.

A mass attached to a spring experiences the force produced by the spring and accelerates in response. Whenever the mass is moving toward the equilibrium position, it speeds up; whenever it is moving away from the equilibrium position, it slows down. Because the force is constantly changing, a graph of the motion looks like a sine curve.

In our system, the air car is the object which moves; it is connected to springs on each end. The total force constant k for this system is the sum of the Hooke's Law constants for the two springs, k_1 and k_2:

$$k = k_1 + k_2\ .$$

By using different springs and by adding mass to the car, it is easy to determine the effect of different force constants and different masses on the period.

For the harmonic oscillator, $F = ma$ gives

$$-k\,x(t) = m\,a(t)\ ;$$

variable	definition	units
T	period, time for one oscillation	seconds/cycle
f	frequency	cycles/second or Hz
ω	angular frequency, $\omega = 2\pi/T$	radians/second
A	amplitude, the maximum displacement from equilibrium	meters
k	spring constant	N/m
m	mass of the object attached to the spring	kg
$x(t)$	displacement of the mass from equilibrium	m
$v(t)$	velocity of the mass	m/s

Table 8.1: List of symbols

A solution of this equation is

$$x(t) = A\cos(\omega t)\ , \tag{8.1}$$
$$v(t) = -\omega A\sin(\omega t)\ , \quad \text{and} \tag{8.2}$$
$$a(t) = -\omega^2 A\cos(\omega t)\ , \tag{8.3}$$

where

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{8.4}$$
$$= 2\pi\sqrt{\frac{m}{k}}\ . \tag{8.5}$$

In the real world, oscillators are subject to friction and the amplitude eventually decreases with time (unless there is some driving force). For friction which is proportional to speed, the decrease in amplitude is an exponential function of time. When the frictional loss is compared to that of a car moving freely on a level track, it is possible to determine whether the losses arise mainly from air film friction or from losses in the springs.

8.2 Measurement of the spring constants

Obtain a set of six springs and label them: 1, 2, 3, 4, 5, and 6. For each spring, do the following:

1. Suspend the spring from the ring stand rod, with a small weight pan hanging on the lower end. Add a small mass to just get the windings separated. Measure the distance from the table top to the bottom of the weight pan with a meter stick.

2. Add an additional mass Δm sufficient to stretch the spring to about 4 to 6 times its normal length, being careful not to stretch it much more than 6 times its normal length. Measure the table to pan distance again.

3. Record the added mass Δm and the increase in length Δy.

For each spring, calculate the spring constant using

$$k = \frac{g\,\Delta m}{\Delta y}\ .$$

Be sure to get your units correct!

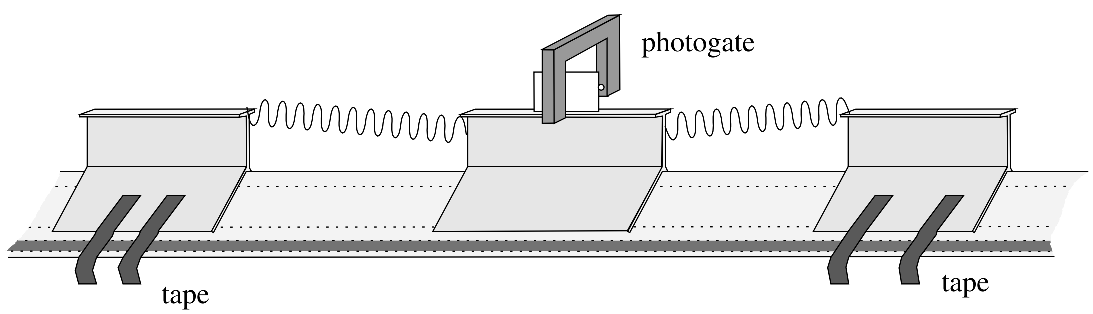
8.3 Period vs. amplitude

1. Clean and level the air track.

2. Select one medium length car and check that the car can carry at least 800 g of additional weight without dragging. Mount a small flag on the car. Record the mass of the car (without any weights); you will need this later.

3. Select two small cars with wire hooks on top.

4. Choose two 3 in. springs and one end of each spring to one end of the car through the holes. Connect to the other ends of the springs to the wire hooks on the two small cars.



5. Tape the skirt of each small car to the track. You can loosen the tape and slide the small cars to adjust the amount of stretching of the springs

6. Pull the large car to the left and right to determine how far the large car can move from its equilibrium position before the spring windings begin to touch: this is the maximum amplitude that may be used; this amplitude needs to be as large as possible, but the springs should never be stretched to more than six times their nominal length as the large car oscillates.

7. Place the photogate assembly at a place where the middle of the flag will block the photogate beam when the car is at rest. It may be easier to adjust the positions of the small cars rather than move the photogate itself.

8. Determine the minimum amplitude which allows the flag to move completely through the beam while moving in each direction.

9. Measure the period for various amplitudes. The amplitudes should range from the minimum to the maximum, with 2 cm increments. Use the `physics` program, menu `Pendulum experiments` to perform the measurements.

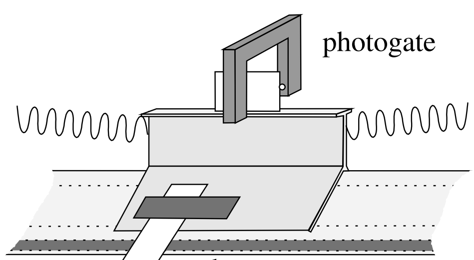
Theory suggests that amplitude does not affect the period. Go to the `Math` menu, submenu `Linear Regression` and perform a linear regression of period vs. amplitude; what is the slope and its error? Is your slope significantly different from zero?

8.4 Damped harmonic motion: effects of friction

1. Prepare a table as follows:

time t (s)	amplitude A (cm)
	10
	9
	8
	7
	6
	5

2. To aid in reading the amplitude, attach a small marker to the large car so that the marker lines up with one of the 10 cm intervals on the ruler.



3. You will also need a stop watch for this measurement.
4. Pull the car slightly more than 10 cm to the right from equilibrium and release it. Start the stop watch.
5. When the maximum amplitude is 10 cm, record the time elapsed.
6. Next, record the time when the maximum amplitude is down to 9 cm, and so forth.
7. When the table is completed, use the `Math` menu, submenu `Linear Regression` to perform a linear regression of $\log(A)$ vs. t. Record the slope and its error.[1]

[1] In this lab, we define $\log(x)$ to be the natural (base e) logarithm of x.

8.5 Period as a function of mass

Now, we will see how changing the mass affects the period.

1. Return to the menu `Pendulum experiments`.

2. Prepare a table as follows:

added mass (g)	total mass m (kg)	period T (s)
200		
400		
600		
800		

3. Now try runs with the above masses added to the car and record your results. Make sure the weights do not interfere with the photogate. In this experiment, the value of the amplitude is not critical.

4. Go to the `Math` menu, submenu `Linear Regression` and perform a linear regression of $\log(T)$ vs. $\log(m)$. What is the slope of the line in your log-log plot? Does your result for the slope agree with Equation (8.5)? In other words, take the logarthm of the Equation (8.5) and find the coefficient of the $\log(m)$ term.

8.6 Period vs. spring constant

Finally, we can explore the effect that changing the springs has on the period.

1. Remove all of the weights from the car.

2. Prepare a table as follows:

nominal length		spring constant	period
left spring	right spring	$k = k_1 + k_2$ (N/m)	T (s)
3 in	3 in		
3 in	2 in		
2 in	2 in		
2 in	1 in		
1 in	1 in		

3. When replacing a spring for a new run, you will need loosen the tape holding the small car. Be sure to adjust the amount of spring stretching by positioning the small car; the springs are not to be stretched to more than six times their nominal length. Position the small cars and the flag so that the middle of the flag will block the photogate beam when the car is at rest.

4. Now, find the period for each of the above spring combinations.
5. Go to the `Math` menu, submenu `Linear Regression` and perform a linear regression of $\log(T)$ vs. $\log(k)$. What is the slope of the line in your log-log plot? Does your result for the slope agree with Equation (8.5)? As before, take the lograithm of the equation.

8.7 Analysis

Most of the analysis has already been carried out during the experiment. However, one crucial calculation still needs to be carried out. You need to take three of your runs where you measured the period and verify Equation 8.5 itself. That is, compare the measured period with the period predicted by Equation 8.5.

In Section 8.4, you found $\log(A)$ vs. t. The slope of this plot is due to friction in the system. If the friction is due mainly to the air film between the car and the track, then the slope should be equal to $-\beta/2$, where β is the friction parameter[2] that we measured in Experiment 4. There, we found $\beta \approx 0.01/\mathrm{s}$. Thus, if the slope of your graph is $\approx -0.005/\mathrm{s}$, air film friction is probably the main source of friction. If your slope is considerably larger, then main source of frction must be due to the springs. What do your data suggest about the major source of friction in this system?

8.8 Group Report

In each part of the lab, we compared the results for the car and springs on the air track with the harmonic oscillator. How well is the car and springs described by the harmonic oscillator?

Some of your results have error estimates. You will be comparing these results with theoretical predictions. Recall from Section 1.2 the meaning of standard deviation. In each case, does your result differ from the predicted value by more than one standard deviation?

You collected a lot of data during this lab. In order to save some work, you do not need to include *all* of the raw data in the lab report: if you include a plot of any results, then you do not need to include associated raw data itself.

[2] The factor of 1/2 is due to the fact that the mass oscillates.

Experiment 9

Resonance and the Velocity of Sound

The objectives for this session are:

1. to study standing waves in a closed tube,
2. to find the dependence of wavelength on frequency, and
3. to measure the velocity of sound in air.

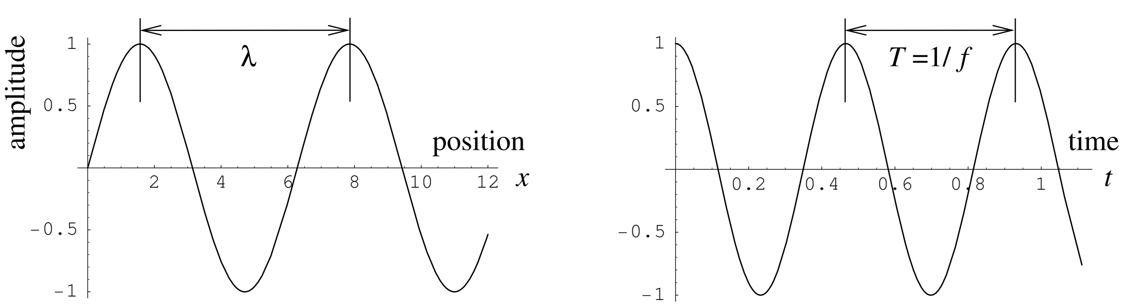
9.1 Introduction

Wave motion can be found almost anywhere: waves in the ocean, gravity waves from exploding stars, radio waves moving through space, seismic waves traveling through the earth, *et cetera*. In this lab, we will study sound waves. Sound waves are, in fact, small changes in the density and pressure of air. These changes in pressure travel through the air much in the same way that waves travel across the surface of the water in a pond. To describe a wave, one can define:

- the wavelength λ, the distance between two maxima of the wave;
- the frequency f, the number of oscillations per unit time; and
- the velocity v, the speed with which the wave travels.

These three quantities obey the relation:

$$v = \lambda f \ . \tag{9.1}$$



In the case of sound waves, the speed depends on the composition of the gas and on the temperature. A model predicts the speed of sound in simple diatomic gases to be

$$v = \sqrt{\frac{1.4RT}{M}}\,, \tag{9.2}$$

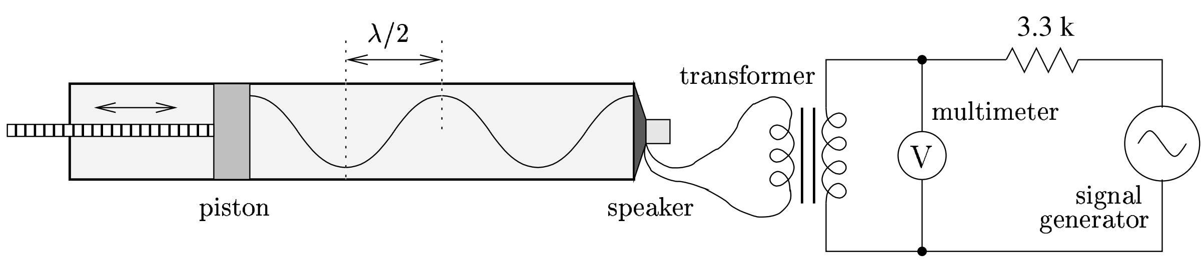
where $R = 8.314\,\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}}$ is the gas constant, T is the temperature in kelvins, and M is the average molar molecular mass. For air, $M \approx 29\,\frac{\mathrm{g}}{\mathrm{mol}}$. According to Equation (9.2), the speed of sound in air at 0° C is about 331 m/s, somewhat faster than commercial jet planes (except the Concorde). The speed at other temperatures can be found using the fact that the speed is directly proportional to the square root of the absolute temperature:

$$v = \left(331\,\frac{\mathrm{m}}{\mathrm{s}}\right)\sqrt{\frac{T}{273\,\mathrm{K}}}\,. \tag{9.3}$$

Sound waves with frequencies between 20 and 20,000 Hz can be detected by the the human ear. Using Equation (9.1), we find, for example, that 600 Hz corresponds to a wavelength of 0.55 m, a distance which can be measured using a meter stick. In this experiment, we will generate sound waves of a known frequency and use resonance effects to measure the associated wavelength. Once we know the frequency and wavelength, we can use Equation (9.1) to determine the velocity v. Theory predicts that the speed is the same for all frequencies.

At one end of the tube, there is a speaker which produces a sound wave in the tube. The speaker is driven by the sine wave output of an oscillator. The sound wave moves through the air in the tube until it meets an obstruction (the piston) and is reflected back again. If the distance between the speaker and the obstruction is chosen properly, the reflected waves add up with the waves newly generated by speaker, giving pressure maxima at certain locations in the tube. These maxima are separated by a distance of $\lambda/2$. This condition of "resonance" occurs when the length of the tube is $\lambda/2$ times an integer.

The resonance condition is detected by the ear as a louder sound. It can also be detected with a multimeter that is connected to the speaker. At resonance, the vibrations in the speaker become larger, causing the the voltage measured by the multimeter to increase. By moving the piston back and forth in the tube, the positions of the resonances can be determined. Twice the distance between two resonances is the wavelength.



9.2 Procedure

1. Turn on the signal generator and the multimeter. Adjust the frequency for the sine wave to some value between 775 and 825 Hz. Record the frequency reported by the display of the

signal generator.

2. Use the meter stick protruding from the end of the tube to slide the the piston in the tube. Support the meter stick with a block of wood so that the meter stick is approximately horizontal; this insures that the reflecting surface is perpendicular to the axis of the tube.

3. You should hear the increased loudness at the location of a resonance.

4. The multimeter must be set to read "AC Volts." Note that the multimeter has a bar-graph display beneath the numerical display. You can locate maxima by finding the greatest length of the bar-graph, then find the precise location by finding the largest numerical reading. If the system is adjusted correctly, the position of the piston at a resonance can be measured to within 1 mm.

5. Scan through the piston positions quickly to determine *approximately* where the resonances occur. Now, go back and measure the positions more accurately. Check that the maxima are roughly equally spaced. Did you miss any of the maxima?

6. Find the distance between the first and last maxima. Divide this quantity by the number of intervening minima to find the distance between *adjacent* maxima. Multiply this result by 2 to obtain the wavelength λ. Multiply the wavelength by the frequency to find the speed of sound for this frequency.

7. Repeat this procedure for four other frequency choices between 600 and 1500 Hz. Your data should now include five different determinations for the speed of sound.

8. Note the air temperature using the thermometer placed on the rear table of the lab.

9.3 Analysis

Your group report should summarize the resonance position data for the several frequencies and the values obtained for the speed of sound.

Examine the five velocity values calculated during the data collection process. Is there any systematic tendency in the values to increase or decrease with frequency? Do you have the impression that some of the measurements were more precise than others? If you assume that all the measurements represent the same speed, to which values would you give more weight in arriving at the most reliable average? Define your best value for the experimental speed of sound from your data.

Convert the Celsius lab temperature to kelvin and use Equation (9.3) to find the expected velocity. Compare this value with the results of your resonance measurements. Assuming that the frequencies were reported by the generator with 0.1% accuracy, and that the distance between first and last maximum (about 800 mm on the average) could be measured to ± 2 mm, the predicted error limit is about 0.3%. Is the difference between the expected speed and the measured speed much larger than this? What sources of error can you suggest?

Experiment 10

Acoustics Demonstrations

The objectives for this session are:

1. to investigate the loudness, pitch, and timbre of sound and
2. to analyze the frequency spectra for various waveforms.

In the previous experiment we used sound to demonstrate general wave properties. In this experiment we will study sound again, but in the context of hearing. Acoustics is concerned with the physiology of hearing, the theory of musical instruments, techniques and equipment for sound recording, design of auditoriums, and many other topics.

Human perception of sound is often described in terms of loudness, pitch, and quality (timbre). These aspects of sensation have corresponding physically measurable parameters.

In this session much of the work will be done as a demonstration for the entire class. Some of the demonstrations require little or no analysis, and this material is reflected in the report as exercises and questions to be answered. Some of the work does require analysis; the report will summarize data to be taken and the method of analysis for these parts.

10.1 Loudness, Range of Intensity

It should be emphasized at the outset that when we deal with human hearing we enter into a realm where the ultimate result, a perception or sensation, is something that occurs in the mind and is not amenable to direct physical measurement. The sensation of loudness involves the physical wave properties of intensity, frequency, and spectral composition, with a generous helping of desirability. That hammering noise from the street crew can be "too loud" even though the actual intensity is less than that of your stereo playing your favorite CD at a loudness somewhat less than you would really prefer. If we wish to know what a sound wave "sounds like," we must ask the listener.

The major determiner of the sensation of loudness is the intensity of the sound wave, although both frequency and spectral composition play an important part. Recall that the intensity of a wave is proportional to the square of its amplitude. For a sound wave, the amplitude is the increase of pressure above atmospheric pressure (or decrease below, although waves may not be symmetrical).

The ear is both exquisitely sensitive and adaptable to a wide range of intensities. Intensity I is defined to be the power per unit area carried by the sound wave. The normal ear in its most sensitive frequency range can detect sounds with intensities as low as $10^{-12}\,\mathrm{W/m^2}$. With a working area less than $0.0001\,\mathrm{m}^2$, this represents a detectable power of less than 10^{-16} W, with the eardrum vibrating by less than an atomic diameter! On the other hand, intensities of up to $1\,\mathrm{W/m}^2$ can be withstood temporarily without damage, giving the ear a dynamic range of 10^{12} between the softest and loudest sounds. There are very few laboratory instruments which are useful over such a wide range. Even the loudest sounds represent very small disturbances in the pressure of the air; $1\,\mathrm{W/m}^2$ corresponds to less than 0.1% fluctuation in pressure about the ambient atmospheric pressure.

With its extremely wide dynamic range, the ear does not respond to sound intensities in a linear fashion. Listeners usually report then the increase in the sensation of loudness is the same when the intensity changes from 10^{-6} to $10^{-5}\,\mathrm{W/m}^2$ as when it changes from 0.001 to $0.01\,\mathrm{W/m}^2$. The sensation of loudness is approximately logarithmic: geometric increases in intensity are interpreted as additive increments. For this reason, it is useful to quantify loudness in terms of the "decibel" scale. The decibel scale derives from the "bel" unit of logarithmic comparison (named after Alexander Graham Bell, a well-known teacher of the deaf). The bel is defined as

$$\text{(The number of bels)} = \log_{10}\left(\frac{I}{I_0}\right) ,$$

where I_0 is a standard value for the intensity. Note that it is mathematically illegal to take the logarithm of a quantity other than a number, so the only way the logarithm of an intensity (or any other quantity with units) can be taken is to divide it by another quantity with the same units. In this case, I/I_0 is dimensionless.

One bel, denoted as a "B," represents an intensity which is 10 times the reference, a rather large increase (equivalent to doubling of the perceived loudness). Consequently, it is more convenient to use the decibel, written "dB," which is defined to be one tenth of a bel.[1] For sound measurements, the reference intensity I_0 customarily used is the above-mentioned faintest sound, $10^{-12}\,\mathrm{W/m}^2$. Such measurements are sometimes designated by dB SPL, where SPL stands for Sound Pressure Level. The conversion between intensity I and decibels SPL is

$$\text{dB SPL} = 10\log\left(\frac{I}{10^{-12}\,\mathrm{W/m}^2}\right) .$$

On this scale, the softest sound is 0 dB and the loudest (safe only for a short time) sound is 120 dB. An increase of 10 dB is equivalent to a ten-fold increase in intensity. An increase of 20 dB is equivalent to a 100-fold increase in intensity. Sitting not too far from a printer or typewriter gives a sound in the range of 60 to 70 dB. Getting close to a jet as it takes off gets dangerously close to 120 dB. A breeze blowing the leaves of the trees is probably around 30 dB.

10.2 Pitch: sounds with different frequencies

The normal ear can detect sounds with frequencies from about 20 Hz to nearly 20,000 Hz. Deep bass notes have low frequencies while treble tones have higher frequencies. The sensitivity of the ear is at a maximum at around 2000 – 3000 Hz; more intensity is required by the ear to detect the highest and lowest frequencies. For loud sounds, between 90 and 100 dB, the sensation of loudness

[1] The prefix *deci-* means one tenth.

is nearly uniform over the audible range of frequencies. The frequency of the sound wave is the primary determiner of sensation of pitch, although spectral composition is also important, and intensity has some effect.

The sensation of pitch is also logarithmic. Listeners tend to identify a doubling of the pitch sensation when the frequency is increased by about a factor of three. There is, however, a strong sense of identity when the frequency is exactly doubled. This is expressed in musical notation by giving the note of twice the frequency the same letter designation as the lower note. Thus, the modern musical scale divides a 2:1 range of frequencies (for example, notes between 220 Hz and 440 Hz) into 12 increments; each note has a frequency that is $\sqrt[12]{2} = 1.0595\ldots$ times the frequency of the next lower note. On a piano keyboard, for example, an octave consists of twelve neighboring keys (white and black). These notes are perceived as equally spaced by the listener. Combinations of notes in chords are sensed differently depending on the combinations. Two notes with 2:1 frequencies (differing by one octave) are are given the same letter name. Simple ratios of frequencies such as 3:2, 4:3, or 5:4 seem to result in pleasing, harmonious sensations when sounded together. Ratios with larger numbers such as 9:8 or 15:16 are regarded as unpleasant, although this, too, depends on the region of frequencies used.

10.3 Timbre, the Quality of Sounds

Sounding the same note at the same loudness on different instruments will give different sensations to the listener. The oboe, violin, and piano offer distinctive and distinguishable tones. This third property of hearing is timbre. A detailed analysis of the pressure fluctuations from different instruments reveals that the shapes of their waves are different. A softly played flute gives a fairly good representation of a sine wave; a reed instrument gives a waveform with more distortions from a pure sine wave.

A mathematical analysis developed by Fourier states that any periodic function can be built up by superimposing sine or cosine waves of frequencies which are integer multiples of the repetition frequency of the function. These integer multiples are called harmonics. In the examples above, the pure sine wave of the flute has only a single frequency, the fundamental or first harmonic. When the shape is distorted from that of a sine wave in the reed instrument, other harmonics are present.

With this analysis, timbre can be expressed in recipes for combining different intensities of the harmonic frequencies. A violin playing middle C has mostly the fundamental 260 Hz, with additional harmonics at multiple frequencies, 520, 780, 1040, 1300, *et cetera*. An oboe playing the same note has more intensity in the higher frequency harmonics.

10.4 Procedure

The instructor will demonstrate some aspects of the hearing process as discussed above. The demonstration may include some or all of the following:

- frequency range of hearing,
- variation of threshold sensitivity with frequency,

- intensity for equal loudness at different frequencies,
- intensity ratio for doubling of loudness sensation,
- pitch intervals with small integer ratios, and
- timbre of a sine wave, triangle wave, square wave, ramp, and pulse.

We can generate a wave by specifying the Fourier coefficients. In the program `physics`, menu `Math`, choose `Plot Fourier series`. You can input coefficients for the different frequencies a_1, a_2, *et cetera*, then plot one cycle of the sums of all the terms,

$$\sum_{k=1}^{n} a_k \sin(2\pi k x) \ ,$$

as a function of x. This facility is useful for checking the signs of the coefficients. The generator will permit trying to generate the square wave with coefficients of different signs.

- Try the coefficients:

k	a_k
1	1
2	0
3	1/3
4	0
5	1/5
6	0
7	1/7
8	0
9	1/9

- See what happens if alternating signs are used instead: 1, 0, $-1/3$, 0, 1/5, 0, $-1/7$, 0, 1/9. Which of these results in a plot which looks like the square wave?
- Repeat this process for the coefficients: 1, 0, $1/3^2$, 0 $1/5^2$, 0, $1/7^2$, 0, $1/9^2$.
- Next, try these coefficients with alternating signs: 1, 0, $-1/3^2$, 0 $1/5^2$, 0, $-1/7^2$, 0, $1/9^2$.
- Finally, try the coefficients: 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7. How would you describe this waveform?

10.5 Applications to Hearing

The relative insensitivity to changes in loudness makes possible group activities. At a lecture or a singing performance, the distances from the source to the closest and most distant listeners may be 1:10, resulting in intensity ratios of 1:100; yet all can hear adequately without discomfort. The ear performs something akin to the Fourier analysis with remarkable sensitivity. The trained ear can detect differences not only between broad classes of instruments, but between nearly identical

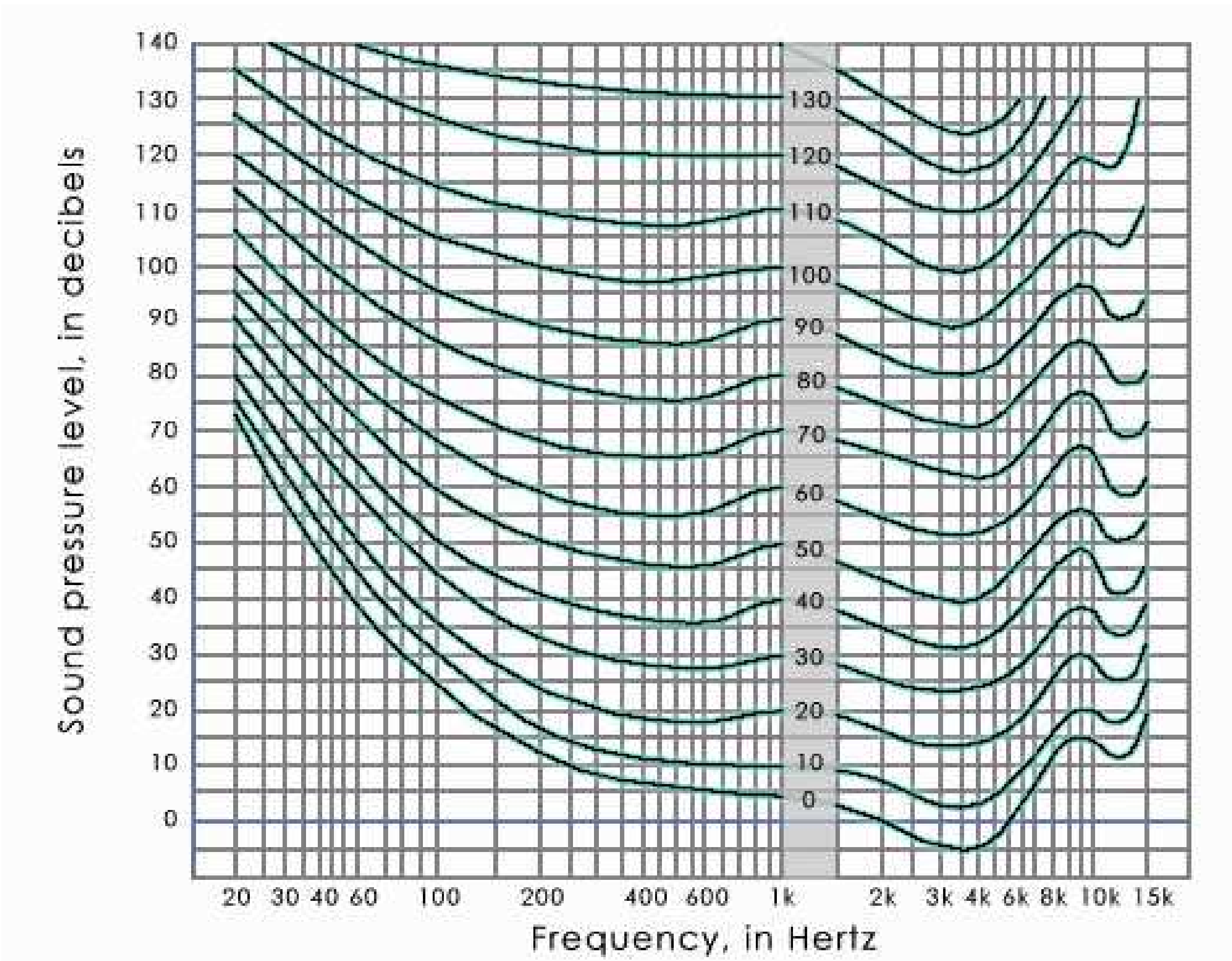


Figure 10.1: Contours of equal loudness vs. intensity and frequency.

instruments. The ordinary listener can identify the voices of dozens of people. Hearing ability seems to decline with age as well as from the effects of disease. In modern society we expect the upper limit of frequencies detected to erode from 20,000 at birth to 5,000 or 8,000 in middle age. There have been reports that the loss of hearing range is much less in societies where loud sounds are rare events. There is now considerable evidence that loud sounds gradually disable the hearing mechanism. Modern society's noise pollution is more than a mere annoyance. The degradation of hearing range has effects which are not obvious, partly because they accumulate so slowly. A person may be able to carry on conversations intelligently, but no longer be able to distinguish voices or musical instruments. When the high frequencies are no longer detected, the analysis of the overtone spectrum is hampered.

An analysis of the sensitivity of hearing has resulted in data summarized as the Fletcher-Munson curves shown in Figure 10.1. These curves summarize the amount of intensity required at each frequency to achieve a particular loudness, as perceived by the ear. For low and high frequencies it is found that much larger intensities are required for moderate loudness sensations. One application of this is to the reproduction of music. At a rock concert, where the intensities are 110 dB if you are getting your money's worth, all the tones are easily detected. If you make a faithful recording of the event and bring it home to enjoy, chances are that your landlady will throw you out if you play it at 110 dB. Instead, you turn down the volume to 90 dB. Now the bass notes seem to have faded: low and high frequencies seem to be suppressed more by turning down the volume. The designers of your sound system have anticipated the difficulty for you: you can turn up the tone control to amplify the bass notes more than the midrange notes, restoring the balance that you enjoyed at the concert. Perhaps a greater concern should be that, if you spend much time at 110 dB concerts, in addition to all the other excitement to which you expose your ears, you may not be hearing much of anything by the time you are forty! It is certainly recommended that unnecessary noises of high intensity be eliminated or absorbed, as with ordinary foam earplugs which drop loudness by 30 dB. You don't really derive that much pleasure from lawn mowers and chain saws. You will derive much more pleasure when you are able to converse with your children and grandchildren.

10.6 Questions

These are to be answered in your report.

1. Calculate the intensity for each sound: 55 dB, 73 dB, 88 dB, and 102 dB. Calculate the dB value for sounds with intensities: 0.001, 0.0002, and 0.00005 W/m^2.

2. Other things being equal, if you want an amplifier which will give a sound output which is twice as loud as your present 20 W amplifier, what amplifier power should you buy? What would be twice as loud as that one?

3. Calculate the 12 frequencies which form an octave of tones ending with A (440 Hz) by dividing repeatedly by the twelfth root of 2. Is the last note 220 Hz? Which tones have ratios which are close to 3:2, 4:3, and 5:4?

Experiment 11

Measuring Absolute zero

The objectives for this session are:

1. to learn about the ideal gas law, and
2. to measure absolute zero.

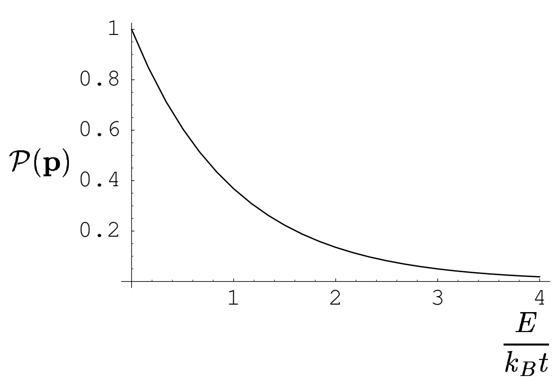
11.1 Introduction

Thermodynamics is the study of behavior of objects in terms of two fundamental quantities, "temperature" and "entropy." In today's lab, we will focus our attention on the meaning of temperature.

We all have some intuitive idea of temperature, what it means when we say that an object is "hot" or "cold." If we add heat to an object, its temperature increases. It wasn't until about 1850 with the work of James Joule that the precise meaning of heat was understood. He showed that heat was simply just another form of energy. He found a universal constant relating mechanical energy and heat energy:

$$\frac{\text{work done}}{\text{heat produced}} \approx 4.18 \, \frac{\text{joules}}{\text{calorie}} \, .$$

Today, we understand that heat is simply the random vibrations of atoms in a solid (one could imagine little harmonic oscillators vibrating back and forth). In a gas, heat is simply the random motions of molecules as they bounce around in a container. The higher the temperature, the faster the molecules are moving. The probability $\mathcal{P}$ that a gas molecule has a certain momentum $\mathbf{p}$ is given by the Gibbs relation:



$$\mathcal{P}(\mathbf{p}) \propto \exp\left(-\frac{E}{k_B T}\right) \tag{11.1}$$

where T is temperature, $k_B \approx 1.3807 \cdot 10^{-23} \frac{\mathrm{J}}{^\circ\mathrm{C}}$ is Boltzmann's constant, and E is the kinetic energy,

$$E = \frac{1}{2} m \mathbf{v}^2 = \frac{\mathbf{p}^2}{2m} \, .$$

This motion of the gas molecules has one effect that is familiar to all of us: pressure. The pressure P exerted by a gas on the walls of a container, measured in units of force per area, is caused by the atoms in a gas bouncing off the wall.

In the ideal gas model, one pictures little particles bouncing around in a container with no interaction between the various particles. Real gases like Oxygen, Nitrogen, Helium, *et cetera* at atmospheric pressure and room temperature behave much like the "ideal gas" model. Using Equation (11.1), one can derive a relation between the temperature T and the pressure P exerted by N molecules on the walls of a container of volume V,

$$PV = N k_B T \, . \tag{11.2}$$

You will note something interesting about Equation (11.2). For a gas in a closed container (constant volume V), the pressure goes to zero in the limit $T \to 0$. Likewise, if you look at Equation (11.1) in the limit $T \to 0$, you can see that the probability vanishes for a molecule to have any momentum other than zero. There is a special temperature, called "absolute zero," at which all vibrations of the molecules in a gas disappear. One can never have a temperature less than absolute zero.

Obviously, absolute zero is a rather special temperature. For this reason, it is convenient to introduce the "kelvin" temperature scale. Zero kelvin is defined to be absolute zero and a change of one kelvin is defined to be equal to change of one degree Celsius. The temperature T in Equation (11.2) is measured in kelvins.

In today's experiment, we will see how the pressure of air changes with as a function of temperature. Then, we will use Equation (11.2) to determine the value of absolute zero in degrees Celsius.

11.2 Procedure

In this experiment, we will change the temperature of air in a fixed volume container and observe the resulting change in pressure. In order to change the temperature, we will add either cold or hot water to the bath. You should collect data for at least five different temperatures. The temperatures should be cover as wide a range as possible (between 0° C and 80° C).

1. For each reading, fill the bath full enough to cover the entire container.
2. Wait for the temperature readings in the bath—along with the manometer readings—to stabilize. You should stir the bath a bit.
3. Record the height of the Mercury in left side and right side of the manometer to the nearest millimeter.
4. Record the temperature of the bath to the nearest tenth of a degree Celsius.

Finally, measure the *inner* diameter of the tube used in the manometer and estimate the volume of the glass sphere as best you can.

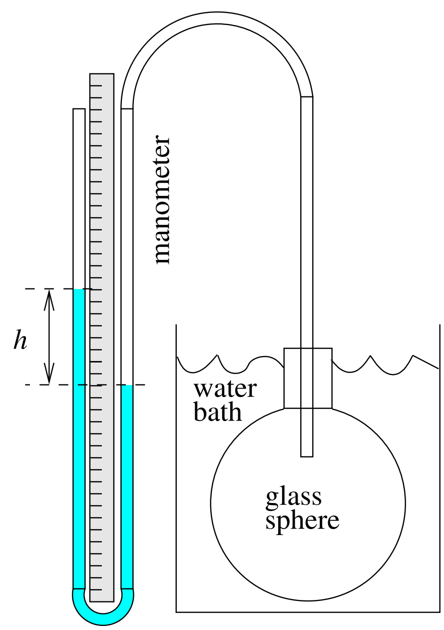


Figure 11.1: Experimental apparatus for measuring absolute zero. The U-shaped tube used to measure the change in pressure is called a manometer.

11.3 Analysis

The manometer measures the *difference* in pressure between the gas in the container and the gas in the room. Thus the total pressure of the gas in the container is

$$P = P_0 + \rho g h \ ,$$

where P_0 is the air pressure in the room and ρ is the density of Mercury in the manometer. The height h is defined in Figure 11.1 and becomes smaller (and negative) with decreasing temperature. Be sure to get your units right!

After finding P for various temperatures, use the `physics` program to perform a linear regression T versus P. The intercept will give you absolute zero. Record slope and intercept, along with their errors.

11.4 Group Report

What value did you get for absolute zero? How does it compare with the official value?

One source of systematic error in this experiment is due to the fact that the volume of gas in the container changes slightly as the mercury in the manometer moves up and down in the manometer during the experiment. In your report, estimate the size of the error produced by this effect. Is it significant or can it be ignored?

Experiment 12

Mechanical equivalent of heat and Heat Capacity

The objectives for this session are:

1. To determine the heat capacity of a solid sample by comparing it to water, and

2. To determine the conversion between mechanical (electrical) energy and heat energy.

12.1 Introduction

Atoms in a solid are constantly vibrating due to their thermal energy. If one part of an object is warmer, the atoms in that part are vibrating with a larger amplitudes. Since the atoms are coupled together, atoms with larger energy will gradually transmit their energy to their neighbors. Thus, heat will flow from hotter regions to colder regions.

An important property of materials is their "heat capacity." This describes how much heat must be added in order to increase the temperature by a fixed amount. If there are more atoms in an object, it will take more heat to raise the temperature. If the atoms can vibrate in more directions (or in more different ways), it will take more heat to increase the temperature. Common units are $\frac{\text{cal}}{g \cdot {}^\circ\text{C}}$ or $\frac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}$. The heat capacity of water in these units is $1.0\,\frac{\text{cal}}{g \cdot {}^\circ\text{C}}$. The amount of heat required to increase the temperature of m grams of water through a temperature increase of ΔT is $mc\,\Delta T$. Removing this quantity of heat from the water reverses the process, decreasing the temperature.

12.2 Heat capacity of aluminum

An ordinary Styrofoam cup will be used as the calorimeter in this experiment. To determine the heat capacity of aluminum, you will fill the cup with about 50 g of water and use a thermometer to monitor the temperature.

Procedure

Fill the cup with 50 g (or 50 ml) of room temperature water. Immerse the thermometer in the water, stirring the water until the temperature reading is stable. Immerse a 1/2 in. aluminum riser block in the beaker of boiling water at the rear of the lab. Take the calorimeter to the rear table;

quickly remove the aluminum block from the boiling water and transfer it to the water in your calorimeter. Stir until the temperature reading attains a maximum value. Note the initial and final temperatures of the water. Dry the metal sample and the cup of the calorimeter. Weigh the sample.

Analysis

Assuming that the heat capacity of water is 1.00 cal/gram-degree, write an equation to set the heat gained by the water equal to the heat lost by the metal sample. Solve for the heat capacity of the Aluminum and compare your value with handbook data.

12.3 Conversion of Electrical to Thermal Energy

The electrical energy supplied during a time interval t to a device at a voltage V and carrying a current I is

$$\text{energy} = VIt \tag{12.1}$$

with units

$$\text{joules} = \text{volts} \cdot \text{ampères} \cdot \text{seconds} .$$

A heating element (a resistor) converts electrical energy to heat energy with 100% efficiency. This conversion of the electrical energy is observed in the environment of the heater as a rise in the temperature of its surroundings. A rise in temperature is often associated with the transfer of heat from a warmer to a cooler object; under these conditions, temperature rise is associated with heat expressed in calories. Because the same amount of energy is being measured with both mechanical units (joule) and thermal units (calorie), the conversion factor, the number of joules in one calorie, can be determined. Electrical heating elements provide one of the most accurate means for obtaining this conversion factor.

Procedure

1. Adjust the power supply so that it outputs about 400 mA.
2. Take the dry calorimeter cup to the electronic balance, weigh it, and record its weight.
3. Use the graduated cylinder to put approximately 60 mL of cold water into the cup.
4. Weigh the cup again and use the change in weight to determine the amount of water in the cup.
5. Place the bulb of 0.1 degree thermometer and the ceramic heater (resistor) in the water in the cup, Stir the contents until a stable starting temperature is observed. Record the starting temperature as T_i.
6. Start the stopwatch at the same time that the power supply is turned on. The ammeter should now be reading a current around 0.4 A.

7. Every one minute record the voltage and the current.

8. After the temperature has risen 5 to 10 degrees, simultaneously turn off the stopwatch and the power supply. Record the heating time in the table. Swirl the calorimeter gently, observing the temperature. Record as T_f the largest temperature reading obtained as the cup is swirled.

9. Dispose of the water in the cup, and dry the heating element.

Analysis

When we studied thermodynamics, we learned that the heat capacity of water is $1.00\,\frac{\mathrm{cal}}{^\circ\mathrm{C}\cdot\mathrm{g}}$. Multiply this by the number of grams of water in the cup and the increase in temperature. The result is the number of calories needed to heat the system through this temperature change. Use Equation (12.1) to calculate the electrical energy that was supplied to the system. The ratio of joules supplied to calories obtained is the conversion factor we are looking for. How does your value compare with that reported by Joule?

12.4 Conclusion

For each of your measurements, you need to estimate the errors associated with your result and compare your result with standard values. Finally, discuss possible sources of error in each experiment.