MAT 405 2004; Homework 1
Due Wednesday, January 21 in class.

Here is a summary of the ideas in the first lecture. The homework problems follow.

Classification

One needs to specify:

- Independant variables. Often this is time t. Can be coordinates x_1, x_2, x_3, $\ldots x_m$.
- Dependant variables. Functions of the independant variable(s). Often written $y(t)$. Can be $y(x_1, x_2, \ldots, x_m)$ and/or there may be several independant variables: $y_1(t)$, $y_2(t)$, $\ldots y_n(t)$.
- Parameters. Quantities that are not functions of the independant variable(s).
- The equation or equations. They can be functions of the dependant variables, independant variables, and parameters.

Differential equations are algebraic equations constructed only from the independant variables, parameters, dependant variables, and derivatives of the dependant variables. These are classified according to:

- **Single equation vs. system of equations.**
- **Differential equation vs. difference equation.** If the independant varible is a integer i, then a dependant variable is often written with a subscript f_i. Derivatives are replaced by the function evaluated at different i: f_{i+1}, f_{i+2}, ...
- **Ordinary differential vs. partial differential**:
  - Ordinary differential: one independant variable.
  - Partial differential: several independant variables.
- **order** The highest derivative found in the differential equations.
- **linear** Each term of the equations have at most one power of any dependant variable. The most general linear differential equation for $y(t)$ has the form:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_2 y'' + a_1 y' + a_0 y = f(t)$$

  where the coefficients a_0, a_1, ..., a_n may be functions of t.
  - **Homogeneous**: $f(t) = 0$. Each term has exactly one power of any dependant variable.
  - **Constant coefficients**. The coefficients a_0, a_1, ..., a_n are not functions of the independant variable.

Note: the terms "order," "partial," and "ordinary" apply only to differential equation.

Assignment

Classify each of the following:

1. Exponential growth, with constant loss, $y(t)$:

$$\frac{dy}{dt} = ky - A$$

2. Forced harmonic oscillator, with $y(t)$:

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \cos(\omega t)$$

3. Duffing's equation for $y(t)$ and $v(t)$:

$$\frac{dy}{dt} = v \quad \frac{dv}{dt} = y - y^3$$

4. Exponential growth equation for P_n:

$$P_{n+1} = kP_n$$

5. Equation of a pendulum, with $\theta(t)$:

$$\frac{d^2\theta}{dt^2} + \frac{b}{m}\frac{d\theta}{dt} + \frac{g}{l}\sin(\theta) = 0$$

6. Harmonic oscillator, with $y(t)$ and $v(t)$:

$$\frac{dy}{dt} = v \quad \frac{dv}{dt} = -qy, \ q > 0$$

7. Equations, with $x(t)$ and $y(t)$:

$$\frac{dx}{dt} = -5x + 2xy \quad \frac{dy}{dt} = -4y + 3xy$$

8. Logistic equation for P_n:

$$P_{n+1} = kP_n(1 - P_n)$$

9. Heat equation in one dimension, for $T(x, t)$:

$$\frac{\partial^2}{\partial x^2}T = \beta\frac{\partial}{\partial t}T$$