Summer Summary of research activities
Summer 20001

E. M. Watson
Geneva College
3200 College Ave., Beaver Falls, PA 15010

To begin preparation for the work ahead, I began the internship with a few days of reading and relatively simple calculations. I studied the limit $x \to 0$ of the 't Hooft model using the Ansatz: $\phi(x) = x^\beta$. Boundaries for β were found through testing at the upper, lower and pole boundaries. An expression for μ, in terms of the constants and β, was easily found at this point. While doing these calculations I also showed the equations symmetry at $x \to 1 - x$. I then moved on to working with the Variational Principle. After doing a few 'simple' book examples, I used Variational Principle on the 't Hooft equation with the Ansatz: $\phi(x) = c_0 x^\beta (1-x)^\beta$. This is when I first saw Beta Functions and Gamma Functions in our work. I then proceeded to use Generalized Eigenfunction forms to solve the equation. This process included use of separate Ansätze for even and odd states and Cholesky Decomposition. As I worked on these calculations with the 't Hooft equation, Jonathan arrived and began work using DLCQ (Discretized Light Cone Quantization) to find similar quantities. Comparison between these independent calculations showed consistency between the methods and verified the results (of course this was after a good bit of initial error correction and code debugging).

As we became more familiar with the methods to be used we worked with a two gluon state. Jonathan and I independently found all of the quantities that I had found for the 't Hooft model and compared results. After discussion with Dr. van de Sande we concluded that the $\phi(x)$ term in the gauge term should be removed leaving only the $\phi(y)$ term. This error was easily corrected within our calculations and we soon finished our work with the two gluon state.

At this point we switched to doing work with *Mathematica* to create bases of states. Jonathan and I created each function separately and then compared examples to find any errors. At this point we made functions to reduce a basis of states using symmetries. We also wrote functions to initially create only the reduced basis.

After completing this sub-project we began discussing and demonstrating the implications of J_z and P_1 values. We read a great deal, from a variety of textbooks, concerning group theory – "...what we are talking about is the decomposition of the direct product of two irreducible representations of the rotation group into a direct sum of irreducible representations." (Griffiths, pg.169) At this point we also worked out a few textbook problems so that we could better understand the concepts of group representation. We also studied the action of unitary operators on states.

After of this preliminary work, we began a study of the "Inchworm Model," an approximation of the meson where only one-gluon states are included. We compared the bound state equations calculated by Simon Dalley (Equations (2.29) and (2.30) of "Mesons on a transverse lattice," hep-ph/0101318) with bound state equations we calculated (Equations (10) through (15) of 'Some notes on the Inchworm Model equation' of http://www.geneva.edu/∼bvds/four/). After a nightmare-ish week or so of calculating, emailing, correcting and re-calculating we found the two sets of bound state equations to be equivalent. (This is all in spite of the fact that *Mathematica* can't seem to simplify statements that are equivalent to zero. Bleh!)

Jonathan and I worked on the boundary conditions of the our version of the bound state equations. We studied the following limits of the wave functions: $x + y \to 0$ of $\psi_S(x, y)$, $x + y \to 0$ of $\psi_A(x, y)$, $x \to 0$ of $\psi(x)$, $x \to 0$ of $\psi_S(x, y)$, $x \to 0$ of $\psi_A(x, y)$, $y \to 0$ of $\psi(x, y)$, and finally $x + y \to 0$ of $\psi_{A'}$. We attempted to do a Variational calculation but found it impossible to solve.

At this point Jonathan and I began working on separate projects. Dr. van de Sande and I independently calculated the general Hamiltonian operator $\langle\psi|H|\psi\rangle$ for mesons on the transverse lattice in the large-N limit and implemented into *Mathematica*. This took days and days to do. (Bleh! again) After more days and days of comparing values and correcting found errors we believe that we came to agreement. While I did this, Jonathan looked at the behavior of α in relation to the coupling constants. He also made attempts to improve on the calculations of $\tilde{f}$ but instead found an error in a previous assumption. This error set us all back a bit and many calculations had to be redone, but all once again came to agreement. Jonathan also created the *Mathematica* functions that created the Hamiltonian matrix using my Hamiltonian operator and numerical values for K, number of particles, z-component of angular momentum, spin, and the coupling constants.

After these steps were completed, Jonathan and I proceeded to do further analysis on the system. I did analysis relating the smallest eigenvalue of a specified Hamiltonian matrix with K and with gluon truncation. I also made probability distribution plots for number of links. Jonathan plotted the structure functions at this time. He also added functions to include charge conjugation symmetry.