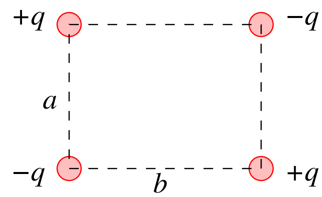
PHY 202 Homework 4

Due Friday, February 20 at SE 227 at 4:30 PM.

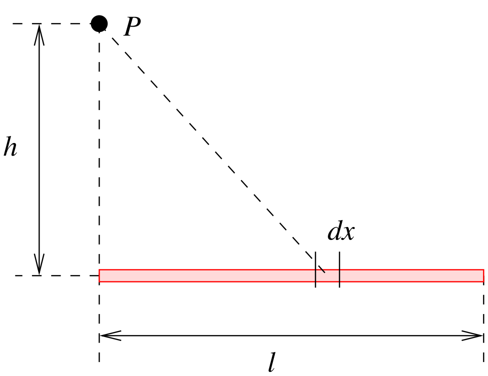
1. Four point charges are located at the vertices of a rectangle as shown.



   Find the potential energy of this configuration of charges. (Hint: Bring the charges in from infinity, one at a time. No work is required to bring in the first charge.) Is the total potential energy positive or negative?

2. Find the Taylor series expansion of $\log(1+x)$ in your calculus textbook (log is the natural logarithm). You will need this for Problems 6 and 4.

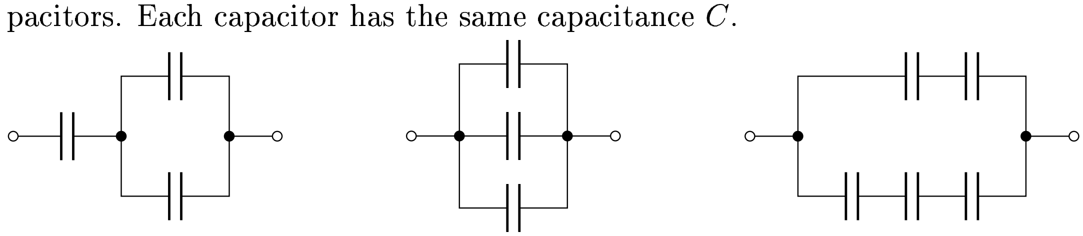
3. A total charge Q is distributed uniformly along a thin rod of length l. Find the electric potential V at a point P that is a distance h away from one end of the rod.



   Hint: Find the electric potential produced by a small segment dx of the rod and integrate over the rod.

4. For Problem 3, find the asymptotic form of the potential in the limit $h \gg l$. Compare your answer with the potential of a point charge.

5. Determine the equivalent capacitance of each of the following networks of capacitors. Each capacitor has the same capacitance C.



6. In class, we derived an expression for the capacitance of a coaxial cable with inner radius r_i and outer radius r_o. Define the distance between the inner and

outer conductors $a = r_o - r_i$ and take the limit $a/r_i \ll 1$. Show that the resulting expression is equivalent to the formula for capacitance of two parallel plates.

7. In class, we derived an expression for the electric potential of a spherical shell of radius R and charge Q. Use this expression to find the associated capacitance. Compute the capacitance of the earth and compare your result to a typical capacitor of about 1 μF.

8. There is a limit to the amount of voltage one can apply to the plates of a parallel capacitor. If the voltage is too large, a spark is produced and current flows freely from one plate to the other.

   What happens? There are always a few ions (charged atoms) in any gas. If one applies an electric field, these ions are accelerated and they eventually hit other molecules. If the electric field is sufficiently large, they can hit other molecules with enough force to produce more ions. These ions then hit other molecules, producing more ions; this chain reaction continues until the gas becomes ionized. Electric current can flow easily through the ionized gas and a spark is produced.

   In *dry* air at room temperature, this electrical breakdown occurs when the electric field exceeds about $3 \cdot 10^6\,\mathrm{V/m}$. For normal conditions, a good rule of thumb is that breakdown occurs at $10^6\,\mathrm{V/m}$. It is interesting to note that breakdown occurs at *smaller* electrical fields if the pressure is lower. At lower pressures, the ions can accelerate more before they hit other molecules.

   I have a Van de Graaff generator with a 25 cm diameter sphere at the top.

   (a) What is the maximum voltage for this generator before a spark is produced?

   (b) What is the capacitance of this sphere?

   (c) How many coulombs of charge can it hold?

   (d) How much energy is stored when it has maximum charge? You must *derive* the formula for the energy of a capacitor.

9. What is the charge of a silicon nucleus? How much voltage must be used to accelerate a proton (radius $1.2 \times 10^{-15}\,\mathrm{m}$) so that it has sufficient energy to just penetrate a silicon nucleus? The radius of a silicon nucleus is about $3.6 \times 10^{-15}\,\mathrm{m}$; assume that it has the potential of a point charge.

10. We found that the electric potential of a point charge Q is

$$V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

    Use this expression and $\mathbf{E} = -\nabla V$ to find the electric field of a point charge.