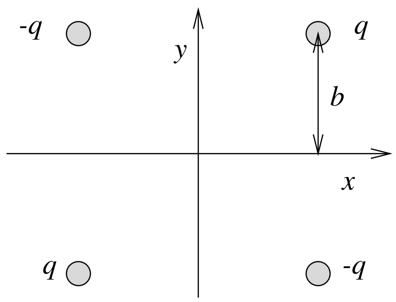
PHY 202 2002; Homework 11

Due Friday, April 25 at SE 227 at noon.

1. In class, we wrote all four of Maxwell's equations in differential form (using $\boldsymbol{\nabla}$). As an exercise in using vectors, write out the equations in terms of x-, y-, and z-components of the vectors. You should obtain eight equations in all: one for Gauß' law, three for Ampére's law, *et cetera*.

2. In an earlier homework, we considered the electric field produced by an "electric quadrupole,"



   Near the center of the square, $|x|, |y|, |z| \ll b$, the electric field has the form:

$$\mathbf{E}(x,y,z) \approx -\frac{3q}{4\pi\epsilon_0 b^3\sqrt{2}}\left(y\hat{x} + x\hat{y}\right) \ .$$

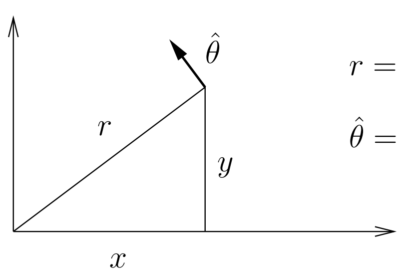
   (a) Verify that $\boldsymbol{\nabla}\cdot\mathbf{E} = 0$.

   (b) Verify that $\boldsymbol{\nabla}\times\mathbf{E} = 0$.

3. The magnetic field for a wire running along the z-axis with current I in the $\hat{z}$ direction is

$$\mathbf{B} = \frac{I\mu_0}{2\pi r}\,\hat{\theta}$$

   where $r \neq 0$ and $\hat{\theta}$ is a unit vector in the "angular" direction.



$$r = \sqrt{x^2+y^2}$$

$$\hat{\theta} = \frac{-y\hat{x} + x\hat{y}}{r}$$

   Thus, we can write $\mathbf{B}$ as

$$\begin{aligned}\mathbf{B}(x,y,z) &= \frac{I\mu_0}{2\pi\sqrt{x^2+y^2}}\cdot\frac{-y\hat{x}+x\hat{y}}{\sqrt{x^2+y^2}} \\ &= \frac{I\mu_0\left(-y\hat{x}+x\hat{y}\right)}{2\pi\left(x^2+y^2\right)} \ .\end{aligned}$$

(a) Find the partial derivatives $\frac{\partial}{\partial x}\mathbf{B}$, $\frac{\partial}{\partial y}\mathbf{B}$, and $\frac{\partial}{\partial z}\mathbf{B}$.

(b) Verify that $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$.

(c) Verify that $\boldsymbol{\nabla}\times\mathbf{B} = 0$.

4. In class, we discussed traveling wave solutions of Maxwell's equations. Such solutions apply to radio waves moving through space, for example. Maxwell's equations also have standing wave solutions. For instance, the electromagnetic fields in a microwave oven are standing waves. Consider the standing wave

$$\begin{aligned} \mathbf{E}(x,y,z) &= E_0 \sin\left(\frac{\pi z}{a}\right)\sin(\omega t)\,\hat{x} \\ \mathbf{B}(x,y,z) &= E_0\sqrt{\mu_0\epsilon_0}\cos\left(\frac{\pi z}{a}\right)\cos(\omega t)\,\hat{y} \end{aligned}$$

between two parallel plates separated by a distance a. The plates are located at $z = 0$ and $z = a$.

(a) Verify that this is a solution of Maxwell's equations.

(b) What is the value of $\mathbf{E}$ at the plates?

(c) Find ω in terms of the separation a.

(d) If the frequency of oscillation is 440 Mhz, what is a?

A faithful man will be richly blessed,
but one eager to get rich will not go unpunished.
Prov. 28:20