PHY 202 2002; Homework 9

Due Wednesday, April 9 at SE 227 by noon.

Faraday's law and inductance can be a bit confusing. For more background on this subject, look in the appropriate chapter of your textbook.

1. Beginning with the equations defining inductance L and resistance R, show that L/R has units of seconds.

2. Verify by substitution that the current $i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$ is a solution of the equation $V - iR - L\frac{d}{dt}i = 0$.

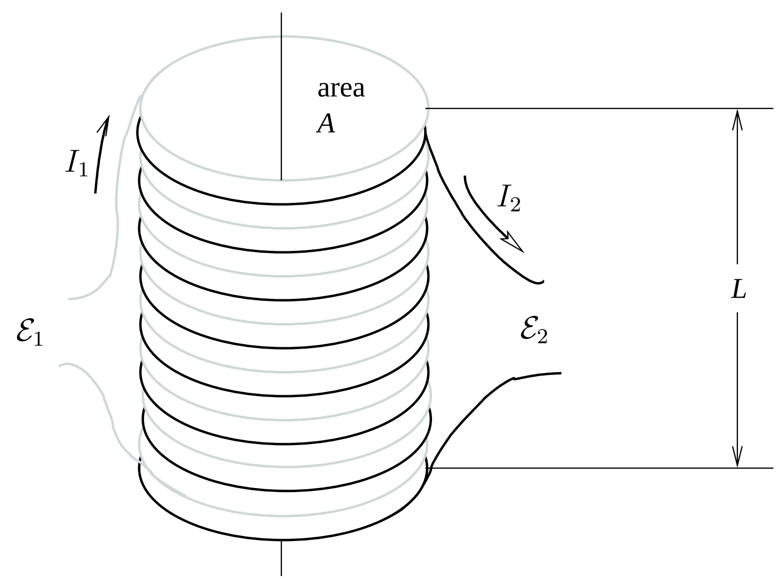
3. A 9 V battery, a 50 Ω resistor, and a 10 H inductor are all connected in series. After the current in the circuit has equilibriated, find

   (a) the power dissapated in the resisitor,

   (b) the power consumed by the inductor, and

   (c) the total energy stored in the inductor.

4. Consider a transformer consisting of two insulated wires wrapped around a cylinder. The cylinder has length L and cross sectional area A. The first wire is wrapped N_1 times and the second wire is wrapped N_2 times.



For simplicity, assume that L, N_1/L, and N_2/L are very large and ignore any "end effects." The first wire is connected to an oscillating voltage source (a wall outlet, for instance) so that $\mathcal{E}_1(t) = V_{\text{in}}\sin(\omega t)$. The second wire is connected to a multimeter. Thus, I_2 is nearly zero.

   (a) Use Faraday's law—integrate with respect to time—to find the resulting magnetic flux $\Phi_{\mathbf{B}}$ passing through *one* loop of the first coil.

(b) Using your result for $\Phi_{\mathbf{B}}$, infer the magnetic field in the interior of the cylinder. A hint is posted outside SE 316.

(c) Use Faraday's law to find the induced voltage $\mathcal{E}_2(t)$ in the second wire.

(d) Write your answer for $\mathcal{E}_2(t)$ in terms of $\mathcal{E}_1(t)$.

Note the following about your answer:

- The $\mathcal{E}_2(t)$ is a function of N_2/N_1.
- $\mathcal{E}_2(t)$ is not a function of the frequency ω or of the geometry, L or A.
- This exercise shows us how transformers can change the voltage of an alternating source. That is, one just changes the ratio N_2/N_1.
- The analysis becomes more complicated when there is also current I_2 in the second wire. For instance, we could connect the second wire to a resistor. However, we would find that the average power $\mathcal{E}_1 I_1$ going into the transformer is always equal to the average power $\mathcal{E}_2 I_2$ going out of the transformer. Energy is conserved!
- Normally, transformers are quite efficient, losing only a percent—or less—of the total power to resistance in the wires.

5. In lecture next week we will briefly discuss the RL circuit. The differential equation governing the RL circuit is analogous to the differential equations we obtained for RC circuits and for an object—like a boat—subject to viscous forces.

   Complete the the following table:

system	differential equation	solution	variables
slow boat			coefficient of friction k, mass m, velocity $v(t)$
RC circuit			
RL circuit			

He who finds a wife finds what is good
and receives favour from the Lord.
Prov. 18: 22