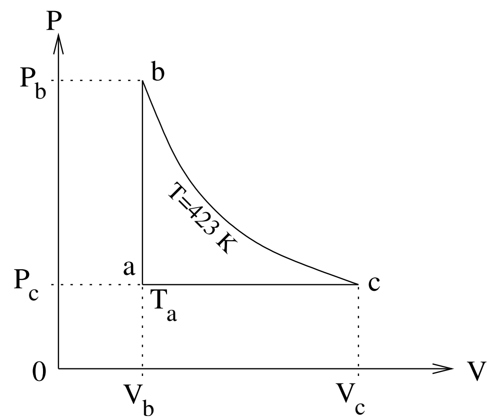
PHY 201 Homework 14 Solution

All problems are from Chapter 20 of the textbook. Problem "24" was supposed to be problem "34"

3 We know that heat obeys $Q_H = Q_L + W$ (energy conservation), and the efficiency of an engine is defined as $e = W/Q_H$. so

$$Q_L = Q_H - W = \frac{W}{e} - W = W\left(\frac{1}{e} - 1\right) = 815\,\mathrm{MW}\ .$$

6 Since this is an engine, the total work is positive, and the cycle must be clockwise.



We are given $T_a = 273\,\mathrm{K}$, $P_c = 1\,\mathrm{atm}$, $T = 423\,\mathrm{K}$, and $n = 1\,\mathrm{mol}$. Using the ideal gas law, $V_b P_c = nRT_a$. Since the curve b-c is isothermal, the ideal gas law tells us $P_b V_b = P_c V_c = nRT$. To find efficiency, the work done and heat transferred must be calculated for each process.

a-b For an isochoric process $\Delta W = 0$ and heat added is $\Delta Q = nC_V(T - T_a)$.

b-c For an ideal gas undergoing and isothermal process, the internal energy U is constant. So

$$\Delta Q = \Delta W = nRT \log\left(\frac{V_c}{V_b}\right) = nRT \log\left(\frac{V_c P_c}{V_b P_c}\right) = nRT \log\left(\frac{T}{T_a}\right)$$

c-a For an isobaric process the work done by the gas is $\Delta W = P_c(V_b - V_c) = nRT_a - nRT = nR(T_a - T)$. The heat added is $\Delta Q = nC_P(T_a - T)$. Note that both of these are negative.

The heat removed from the system is only from the isobaric process:

$$Q_L = nC_p(T - T_a)\ .$$

The heat added (don't count the isobaric part) is

$$Q_H = nC_V(T - T_a) + nRT \log\left(\frac{T}{T_a}\right)\ .$$

Thus, the efficiency is

$$e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{nC_p(T - T_a)}{nC_V(T - T_a) + nRT\log\left(\frac{T}{T_a}\right)} .$$

For monatomic ideal gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$ and

$$e = 1 - \frac{\frac{5}{2}(T - T_a)}{\frac{3}{2}(T - T_a) + T\log\left(\frac{T}{T_a}\right)} = 8.59\%$$

It is interesting to note that the efficiency only depends on the ratio T/T_a,

12 For a Carnot cycle, the efficiency is $e = 1 - T_L/T_H$ so $T_H = T_L/(1-e)$ where $e = 35\%$ and $T_L = 360° = 633\,\mathrm{K}$. Using $e' = 1 - T_L'/T_H$, the temperature needed for $e' = 50\%$ efficiency is

$$T_L' = T_H(1 - e') = T_L\frac{1 - e'}{1 - e} = 633\,\mathrm{K}\frac{1 - 0.5}{1 - 0.35} = 486\,\mathrm{K} = 213° .$$

16 We have $T_H = 580° + 273 = 853\,\mathrm{K}$. The efficiency is, for a Carnot engine

$$e = \frac{\Delta W}{\Delta Q_H} = 1 - \frac{T_L}{T_H} \qquad (1)$$

where the work is $\Delta W = 570\,\mathrm{kW}$ and $Q_H = 1350\,\frac{\mathrm{kcal}}{\mathrm{s}}$. We want to find the temperature T_L.

$$T_L = T_H\left(1 - \frac{\Delta W}{\Delta Q_H}\right) = 853\,\mathrm{K}\left(1 - \frac{570 \times 10^3\,\frac{\mathrm{J}}{\mathrm{s}}}{1350 \times 10^3\,\frac{\mathrm{cal}}{\mathrm{s}} \cdot 4.186\,\frac{\mathrm{J}}{\mathrm{cal}}}\right) = 767\,\mathrm{K} = 494°$$

24 (typo)

$$C_p = \frac{T_H}{T_H - T_L} = \frac{258\,\mathrm{K}}{295\,\mathrm{K} - 258\,\mathrm{K}} = 6.97$$

26 Since a refrigerator is a heat engine running backwards, we can use Equation (1) above, where $T_H = 22° = 295\,\mathrm{K}$. Solving for the work,

$$\Delta W = \Delta Q_H\left(1 - \frac{T_L}{T_H}\right)$$

When $T_L = 0° = 273°$, $\Delta W = 209\,\mathrm{J}$. When $T_L = -15° = 258°$, $\Delta W = 351\,\mathrm{J}$.

27 The coefficient of performance is $\frac{Q_H}{W} = \frac{1}{e} = 2.86$.

34 (typo) Let $L = 79.7\,\frac{\mathrm{kcal}}{\mathrm{kg}}$ be the heat of fusion and $m = 1\,\mathrm{m}^3 \cdot \frac{10^3\,\ell}{\mathrm{m}^3} \cdot \frac{1\,\mathrm{kg}}{\ell} = 10^3\,\mathrm{kg}$ be the mass of the water. The change in entropy of the ice is

$$\Delta S = \frac{mL}{T} = -\frac{10^3\,\mathrm{kg} \cdot 79.7\,\frac{\mathrm{kcal}}{\mathrm{kg}}}{273\,\mathrm{K}} = -292\,\frac{\mathrm{kcal}}{\mathrm{K}}$$

Since the water is losing heat, its entropy decreases.

35 The heat gained by the ice is also mL, but the temperature is now $T = 263\,\mathrm{K}$. The entropy of the ice increases by

$$\Delta S = \frac{mL}{T} = \frac{10^3\,\mathrm{kg} \cdot 79.7\,\frac{\mathrm{kcal}}{\mathrm{kg}}}{263\,\mathrm{K}} = 303\,\frac{\mathrm{kcal}}{\mathrm{K}}$$

The total change in entropy is $303\,\frac{\mathrm{kcal}}{\mathrm{K}} - 291\,\frac{\mathrm{kcal}}{\mathrm{K}} = +11.1\,\frac{\mathrm{kcal}}{\mathrm{K}}$.

37 This is essentially the example of two heat baths in contact that we did in lecture. $T_L = 27^\circ = 300\,\mathrm{K}$, $T_H = 240^\circ = 513\,\mathrm{K}$, and $\Delta Q = 7.50\,\frac{\mathrm{cal}}{\mathrm{s}}$. We have,

$$\Delta S = -\frac{\Delta Q}{T_H} + \frac{\Delta Q}{T_L} = +0.0104\,\frac{\mathrm{cal}}{\mathrm{K} \cdot \mathrm{s}}$$