PHY 201 Homework 13 Solution

This is the solution for Problem 50 in Chapter 19. Many students confused the number of degrees of freedom with the number of moles. The textbook (unfortunately) uses n for both of these.

Let n be the number of degrees of freedom for each molecule. By the *equipartition theorem* (see page 500), the average energy for a molecule is $\frac{n}{2}k_BT$. If there are N particles, the total internal energy is $U = N\frac{n}{2}k_BT$.

Let us convert this to moles. Let m be the number of moles, then $Nk_B = mR$. (Recall the two forms of the ideal gas law, $PV = mRT$ and $PV = Nk_BT$.) Thus,

$$U = \frac{n}{2}mRT \ . \tag{1}$$

The definition of heat capacity at constant volume is

$$\Delta Q = mC_V\Delta T \ . \tag{2}$$

At constant volume $\Delta W = 0$ so, using the first law of thermodynamics, $\Delta Q = \Delta U$. Combining Equation (1) and (2),

$$mC_V\Delta T = \frac{n}{2}mR\Delta T \tag{3}$$

so we get $C_V = \frac{n}{2}R$.

Since, in general, $C_P = C_V + R$ (see Equation (19-9) in the textbook),

$$C_P = \left(\frac{n}{2} + 1\right)R = \frac{n+2}{2}R \tag{4}$$