PHY 201 Homework 11 Solution

1. Given Ansatz:

$$\begin{aligned} y_1(x,t) &= A\sin(kx-\omega t) & (1)\\ y_2(x,t) &= A\sin(kx+\omega t)\ , & (2) \end{aligned}$$

show these are solutions of the wave equation

$$v^2 \frac{\partial^2}{\partial x^2}\, y - \frac{\partial^2}{\partial t^2}\, y = 0\ . \qquad (3)$$

Which is going to the left and which is going to the right?

Solution: First, find derivatives:

$$\begin{aligned} \frac{\partial}{\partial x}\, y_1 &= kA\cos(kx-\omega t)\\ \frac{\partial^2}{\partial x^2}\, y_1 &= -k^2 A\sin(kx-\omega t) = -k^2 y_1\\ \frac{\partial}{\partial t}\, y_1 &= -\omega A\cos(kx-\omega t)\\ \frac{\partial^2}{\partial x^2}\, y_1 &= -\omega^2 A\sin(kx-\omega t) = -\omega^2 y_1 \end{aligned}$$

Substitute into the wave equation (3),

$$\begin{aligned} v^2(-k^2 y_1) - (-\omega^2 y_1) &= 0\\ y_1(-v^2k^2+\omega^2) &= 0 \end{aligned}$$

Thus, we have the relation $\omega^2 = v^2k^2$. Therefore, y_1 is a solution of the wave equation, if $v = \omega/k$.

Verification of y_2 proceeds in a similar manner.

Finally, y_1 is moving to the right, and y_2 is moving to the left.

2. Given Equations (1) and (2) above, show that $y_1 + y_2$ makes a standing wave.

$$y_1 + y_2 = A\left(\sin(kx-\omega t) + \sin(kx+\omega t)\right)$$

Use the trig identity, $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$,

$$\begin{aligned} y_1 + y_2 &= A\left(\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t) + \sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t)\right)\\ y_1 + y_2 &= 2A\sin(kx)\cos(\omega t) \end{aligned}$$

This is a standing wave.

3. Textbook, chapter 15, problem 37: 440 Hz, 880 Hz, 1320 Hz, 1760 Hz.

4. Textbook, chapter 15, problem 40: Distance between nodes is

$$\frac{\lambda}{2} = \frac{v}{2f} = \frac{270\,\frac{\mathrm{m}}{\mathrm{s}}}{2\cdot 131\frac{1}{\mathrm{s}}} = 1.03\,\mathrm{m}$$