PHY 201 Homework 11

Due Friday, December 6 at noon.

1. Consider the three functions:

$$\begin{aligned} y_1(x,t) &= A_1 \sin(kx - \omega t + \phi_1) \\ y_2(x,t) &= A_2 \sin(kx + \omega t + \phi_2) \\ y_3(x,t) &= C \cos(kx) \cos(\omega t) \end{aligned}$$

   Thus, y_1 is a traveling wave moving in the positive x direction, y_2 is a traveling wave moving in the negative x direction, and y_3 is a standing wave. In this problem, you will show that a standing wave can be written as the sum of two traveling waves.

   (a) What are A_1 and ϕ_1 called?

   (b) Show that y_1, y_2, and y_3 are all solutions of the wave equation

$$v^2 \frac{\partial^2}{\partial x^2} y(x,t) - \frac{\partial^2}{\partial t^2} y(x,t) = 0 ,$$

   when $v = \omega/k$.

   (c) Use trig identities to write y_1 in the form:

$$y_1(x,t) = (\textit{something}) \cos(kx - \omega t) + (\textit{something else}) \sin(kx - \omega t) \ .$$

   Do the same thing for y_2.

   (d) Next, use trig identities to write y_1 in the form:

$$y_1(x,t) = A_1 \cos(\phi_1) \sin(kx) \cos(\omega t) + \cdots \ .$$

   Do the same thing for y_2.

   (e) Finally, equate $y_1 + y_2$ with y_3. Compare coefficients of like terms to find A_1, A_2, ϕ_1, and ϕ_2 such that y_3 can be written as a sum of y_1 and y_2.

   You have now shown that a standing wave can be written as a superposition of two traveling waves.

2. Consider two traveling waves,

$$\begin{aligned} y_1(x,t) &= A \cos(kx - \omega t) \\ y_2(x,t) &= A \cos(kx + \omega t) \ . \end{aligned}$$

   When added together, they form a standing wave. Find the positions of the nodes of this standing wave (draw a graph illustrating you answer). What is the wavelength and the frequency of the standing wave?

3. A single pulse can be written in terms of a Gaussian function. Consder the wave:

$$y(x,t) = Ae^{-(kx+\omega t)^2/2}$$

where $k = 2.1\,\frac{1}{\mathrm{m}}$ and $\omega = 3.6\,\frac{1}{\mathrm{s}}$.

   (a) Plot the function $y(x, 0)$ as a function of x. Use graph paper to do this.

   (b) Plot the function $y(x, 1\,\mathrm{s})$ as a function of x.

   (c) Verify that this function is a solution of the wave equation.

   (d) How fast is this wave moving? Is it moving to the right or to the left?

4. The note middle "A" is defined to be 440 Hz. Find the wavelength and period, in air, of this note.

5. A cello string, playing a low "A," vibrates with a frequency of 220 Hz. The vibrating part of the string is 70 cm long and has a 1.2 g total mass.

   (a) Find the tension in the string.

   (b) Find the frequency of the harmonic where the string vibrates with three vibrating segments.

6. When the speed of sound in air is 340 m/s, a pipe organ plays middle "A" at the correct frequency. On a very warm day, the speed of sound goes up to 346 m/s due to the increased air temperature. If a piano plays middle "A" (at the correct frequency) and the organ plays middle "A," what is the beat frequency that is heard? You will have to read in your textbook about "beats."