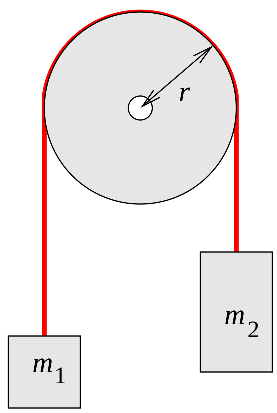
PHY 201 Homework 9

Due Friday, November 8 at SE 316 at noon.

The second midterm will be held on Wednesday, November 13. It will be based on homeworks 6 through 9. There will be a physics tea held on Wednesday, November 6, 8:00–10:00 PM.

1. 

   Two weights, masses m_1 and m_2, are suspended by a rope from a pulley. The mass of the pulley and any friction are negligible.

   (a) Assuming $m_2 > m_1$, find the acceleration of m_2.

   (b) What happens in the limit $m_2 \gg m_1$? Explain.

   (c) What happens in the limit $m_1 \to m_2$? Explain.

2. Now, let us assume that the pulley has a substantial mass which can be approximated as a uniform disk (or cylinder) of iron, total mass M and radius r.

   (a) What is the moment of inertia of the pulley? (See the lab manual.)

   (b) What are the torques acting on the pulley? Note: the two segments of the rope no longer have the same tension!

   (c) How is the angular acceleration α of the pulley related to the acceleration a of the two weights?

   (d) Find the angular acceleration α of the pulley.

   (e) What happens to α in the limit $M \gg m_1, m_2$?

   (f) Make up some reasonable numbers for r, m_1, m_2, and M and find the acceleration of m_2.

3. Consider two particles of mass m_1 and m_2 with separation R.

   (a) Choose a coordinate system such that the center of mass is at the origin.

   (b) Find the components of the moment of inertia for rotations about the center of mass.

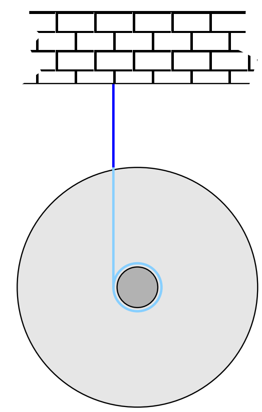
   (c) What is the total 'orbital' angular momentum of the earth and moon revolving around each other? Approximate the earth and moon as point particles.

   (d) What is the distance from the center of the earth to the center of mass? Compare this to the radius of the earth.

   (e) One side of the moon always faces the earth. What is the angular momentum of the moon as it rotates about its axis?

4. A uniform solid sphere rolls without slipping down an inclined plane at 15° with respect to the horizontal. Find the speed and displacement of the ball 2 seconds after it starts moving. What fraction of the ball's kinetic energy is rotational?

5. Consider a yo-yo with a 0.635 cm diameter shaft and total diameter of 3.175 cm and mass 120 g. In this problem, ignore the thickness of the string itself.
In your solutions, put in numbers at the last possible step.



   (a) If you approximate the yo-yo as a uniform disk, what is the moment of inertia of the yo-yo spinning about its axis?

   (b) The yo-yo has both rotational and translational motion. How are these two related?

   (c) When the yo-yo is released from rest, how much time does it take for 1 m of string to unwind?

   (d) When the yo-yo is descending, what is the ratio of rotational to kinetic energy?

6. Consider a uniform, thin metal rod with mass M and length a. Choose a coordinate system such that the rod lies on the y-axis with one end at the origin.

   (a) Let $\lambda(y)$ be the mass per unit length of the rod. What is λ?

   (b) In lecture, we defined the center of mass $\mathbf{R}_{cm}$ of n particles, masses m_i and positions $\mathbf{r}_i$. In the limit $n \to \infty$, the center of mass can be written as an integral:

$$\mathbf{R}_{\mathrm{cm}} = \lim_{n\to\infty} \frac{1}{M} \sum_{i=1}^{n} m_i \, \mathbf{r}_i = \frac{\hat{y}}{M} \int \lambda(y) \, y \, \mathrm{d}y \, .$$

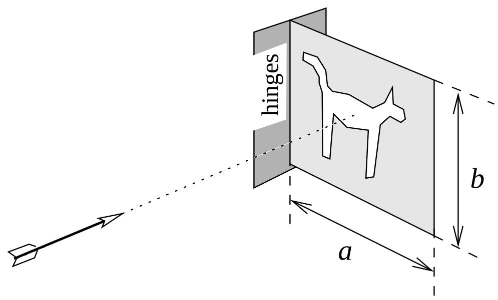
   Use this integral to find the position of the center of mass.

   (c) In the same manner, we can write the formulas for the moment of inertia tensor in integral form. (Here, r_i is the distance from the z-axis.) Turn the following sum into an integral:

$$I_{xx} = \lim_{n\to\infty} \sum_{i=1}^{n} m_i \left(y_i^2 + z_i^2\right) \, .$$

Use this to find the moment of inertia tensor $\mathbb{I}$. ($\mathbb{I}$ will be diagonal for this problem.)

7. Mr. Lance is practicing for hunting season; He sets up a target on a hinged gate, as shown. The gate swings freely on its hinges.



He is no good at shooting deer, but he is an expert marksman when it comes to *Felis catus*. The gate has width $a = 1\,\text{m}$, height $b = 1.2\,\text{m}$ and a mass of $5\,\text{kg}$. Eric's arrow, mass $500\,\text{g}$, flies through the air at a speed of $100\,\text{m/s}$ and hits the target right at the center of the gate and the gate starts turning. (For solving this problem, you will find it convenient to approximate the arrow as "point particle.") The moment of inertia of the gate as it rotates on its hinge is $I = Ma^2/3$.

   (a) Is angular momentum conserved in this problem?

   (b) Is rotational energy conserved? Hint: is this an elastic or inelastic collision?

   (c) Find the rotational velocity of the gate just after the arrow hits it.

8. A uniform solid sphere and a cylinder, each of mass M and radius R have a race, starting from rest, and rolling down an inclined plane. Which of the two wins? Explain why.

9. In class this week, we will do a gyroscope demonstration. One can approximate the gyroscope as a $4\,\text{cm}$ radius hoop (most of the mass is at a radius of $4\,\text{cm}$ from the axis of rotation) with a total mass of $1.4\,\text{kg}$. Let's say that I use a 20 Watt electric motor to speed up the gyroscope.

   (a) Find the moment of inertia of the gyroscope as it rotates about a fixed axis.

   (b) Ignoring friction, what is the rotational velocity of the gyroscope after 30 seconds of acceleration? (Hint: think about energy.)

   (c) What is the final angular momentum $\mathbf{L}$? Draw a nice big picture and explain the direction of $\mathbf{L}$.

   (d) Now, what torque $\boldsymbol{\tau}$ would I need to apply to the gyroscope to get it to precess at a rate of $0.1\,\text{rad/sec}$? Find its magnitude and indicate an appropriate direction for $\boldsymbol{\tau}$ in your nice big picture.

(e) Construct an example of a force **F** that would produce this torque. Indicate where you would apply this force and indicate the direction of **F** in your nice big picture.