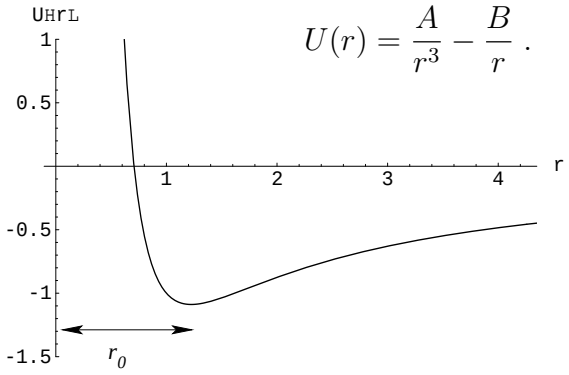
PHY 201 Homework 8

Due Friday, October 25 at SE 316 at noon.
There will be a physics tea on Wednesday, October 23, at 3510 5th Ave. between 8 and 10 pm.

1. Two particles having the same mass undergo an *elastic* collision. If the initial velocity of one of the particles was zero, show that the angle between the two particles after the collision is 90°. Hint: what does 90° imply about the dot product of two vectors?

2. Express the kinetic energy of an object, mass m, as a function of momentum $\mathbf{p}$.

3. The binding energy of an electron and an ion as a function of separation r is sometimes modeled by the function



$$U(r) = \frac{A}{r^3} - \frac{B}{r} .$$

It is useful to choose a coordinate system such that the ion lies at the origin and the electron lies on the x-axis.

   (a) Draw a picture defining your coordinates and show that the force acting on the electron is

$$\mathbf{F} = \left(-\frac{3A}{r^4} + \frac{B}{r^2} \right) \hat{n}$$

   where $\hat{n}$ is a unit vector pointing in the direction of the ion.

   (b) Find the equilibrium separation r_0 and the corresponding "binding energy" $U(r_0)$ as a function of A and B.

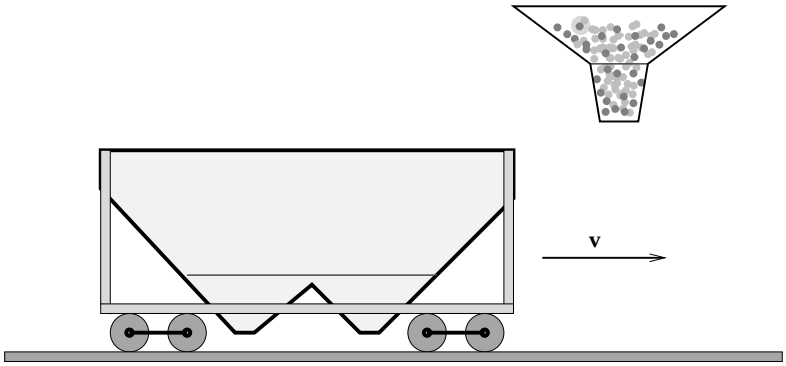
   (c) Find the spring constant for small oscillations about the equilibrium.

4. Conservation of momentum can also be helpful for problems where the mass is changing. In such cases it is useful to rewrite Newton's second law as

$$\mathbf{F} = \frac{d}{dt}\mathbf{p}$$

Use this equation to derive $\mathbf{F} = m\mathbf{a}$.

5. A 5000 kg empty freight car rolls without friction under a stationary hopper filled with coal. The car is 15 m long.



The car approaches the hopper with a steady speed of 2.5 m/s. As the car passes beneath the hopper, the hopper fills the car with coal at a constant rate. A total of 10,000 kg of coal is loaded.

(a) The added weight slows down the freight car. What is the final speed of the freight car?

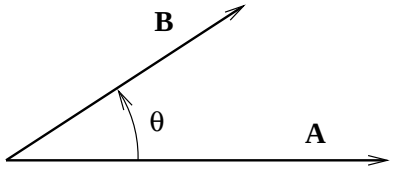
(b) Now, imagine a locomotive pushes the freight car at a constant velocity of 2.5 m/s under the hopper, find the force needed to keep the freight car moving at a constant velocity. Hint: use the equation in problem 4.

(c) What was the work done by the locomotive?

(d) What is the total kinetic energy of the coal afterwards?

You can find a discussion of this kind of problem in most textbooks.

6. Two vectors **A** and **B** lie in the xy-plane with an angle θ between them:



Using the five properties of the cross product introduced in lecture, prove[1] that

$$\mathbf{A} \times \mathbf{B} = ||\mathbf{A}||\,||\mathbf{B}||\,\sin(\theta)\,\hat{z}\ .$$

Hint: introduce a coordinate system.

[1] Be sure to justify each step.

Whatever your hand finds to do, do it with all your might, for in the grave, where you are going, there is neither working nor planning nor knowledge nor wisdom.
Ecclesiastes 9:10