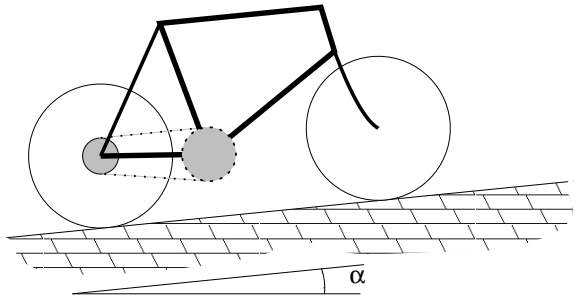
PHY 201 Homework 6

Due Friday, Oct. 11 at SE 316 at noon.

The problems in this homework set involve a lot of work; get started on them right away.

1. 

I ride my bicycle up a 10% incline at a constant speed for a total distance of 10 miles in one hour. Let $m = 75\,\text{kg}$ be the total mass of me and my bicycle. A 10% incline means that $\tan(\alpha) = 0.1$. In the following, ignore any friction due to the air.

   (a) What is the frictional force exerted by the bicycle on the road? (magnitude and direction)

   (b) What is the work done by the road on the bicycle?

   (c) What is the work done by the bicycle rider?

   (d) What is the average power output of the bicycle rider?

2. A 6 kg cat falls off the top of a tall building.

   (a) What is the work done by gravity after the cat has fallen for 1.80 seconds?

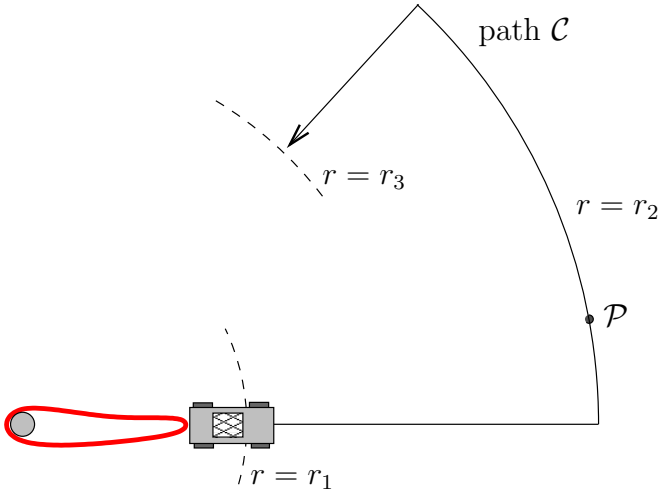
   (b) What was the *average* power from gravity after 1.80 seconds?

   (c) What was the *instantaneous* power from gravity at this time?

   (d) What happens when the cat hits the ground?

   (e) Express the velocity v of the cat as a function of the distance fallen and g. Use this to find an expression for the work done by gravity as an function of v and mass m.

3. I attach a rubber band to a fixed post and attach the other end to a toy car as shown,



I define polar coordinates (r, θ) with origin at the post. Then I drive the car along path $\mathcal{C}$, starting at radius $r = r_1$ and ending at radius $r = r_3$. Let $\mathbf{F}(r)$ be the force of the rubber band acting on the car.

(a) Find a general expression for W, the total work done by the rubber band on the car.

(b) Show that $W = 0$ in the case $r_1 = r_3$.

(c) Find a numerical value for W when

$$\mathbf{F}(r) = -\left(100\,\frac{\mathrm{N}}{\mathrm{m}^2}\right) r^2 \hat{r}\ ,$$

$r_1 = 0.1\,\mathrm{m}$, $r_2 = 0.5\,\mathrm{m}$, and $r_3 = 0.3\,\mathrm{m}$.

(d) How does W behave as a function of r_2? (If this is not clear to you, try different numerical values of r_2.)

(e) If the car moves along the path $\mathcal{C}$ at a constant speed of $0.4\,\frac{\mathrm{m}}{\mathrm{s}}$. Find the power exerted by the rubber band at the beginning when $r = r_1$. What is the power when the car is at point $\mathcal{P}$? Find the power at the end of the journey when $r = r_3$.

4. Consider a path $\mathcal{C}$ defined in the xy–plane by the parametric equation

$$\mathbf{R}(u) = \rho\cos(u)\hat{x} + \rho\sin(u)\hat{y}$$

with endpoints at the positions $(\rho, 0)$ and $(-\rho, 0)$.

(a) Sketch a graph of this curve.

(b) Find $\dfrac{d}{du}\mathbf{R}(u)$

(c) Consider a force which we define to be $\mathbf{F} = ky\,\hat{y}$. At a point $\mathbf{R}(u)$ on the path, this force is equal to $\mathbf{F}(u) = k\rho\sin(u)\hat{y}$. Why is this?

(d) Evaluate the line integral

$$\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{R}$$

using the equivalent expression

$$\int_a^b \mathbf{F}(u)\cdot\frac{d\mathbf{R}}{du}\,du$$

What are the values for a and b?

(e) What is the answer for (4d) if the endpoints of the interval are $(\rho, 0)$ and $(0, \rho)$ instead?