PHY 201 Homework 1
Due Friday, Sept. 6, by noon at SE 316

1 Introduction to Measurement

This problem is meant to be a warm-up for the first lab next week. The measurement of various quantities is a central theme in experimental physics. There are two basic kinds of measurements:

Discrete measurements, where one counts something: electrons, photons, pions, protons, gummy bears, neutrinos, ... or—in this case—coins.

Continuous measurements, where one measures a length, weight, time, volume, voltage, or something like that. Most of the things that we measure in our labs will be of this type.

In this exercise, we will perform a "counting experiment" and, in the context of this experiment, discuss how errors arise and show why errors often have a Gaussian distribution.

For this exercise, you will need six coins, a piece of graph paper, 2 colored pencils, and a calculator. You will be doing the following: tossing the six coins, counting the number that land "heads," and histogramming the result. On the graph paper, draw the horizontal axis with label h=*number of heads* and the vertical axis with label *number of tosses*. You will need enough room in the vertical direction to record at least 30 measurements. On the horizontal axis, the possible outcomes, 0, 1, 2, 3, 4, 5, and 6, should be spaced as far apart as possible. To get a sufficiently large data sample, you will need to toss the six coins at least 80 to 100 times. It takes a while, but I assure you that I tried this myself! Each student should toss their own coins (sorry, no teamwork allowed). When you are done, histogram the number obtained for each h on your graph and record the total number of times you tossed the coins.

Let us compare your results with those predicted from elementary probability theory. If you toss one coin, there are two possible outcomes: "heads" or "tails." Each of the two possible outcomes has a probability of 50%. If you toss two coins, there are 4 possible outcomes: tails-tails, heads-tails, tails-heads, or heads-heads; the *number of heads* is 0, 1, 1, and 2, respectively. Thus, the probability of obtaining h heads is: 25% for $h = 0$, 50% for $h = 1$, and 25% for $h = 2$. In general, if you toss n coins there are 2^n possible outcomes and the probability of obtaining h heads is

$$\text{Probability} = \binom{n}{h} \frac{1}{2^n} = \frac{n!}{h!\,(n-h)!} \frac{1}{2^n}$$

($\binom{n}{h}$ is the number of 'combinations' of n distinct objects taken h at a time). Using this equation and your total number of tosses (80 to 100), find the number of tosses expected for each value of h. Mark on the graph (with a colored pencil) the expected number of tosses for each value of h.

Next, calculate, $\bar{h}$, the average *number of heads* you obtained using the formula for "mean value" in the lab manual. Note that there is a trick involved: you don't have to explicitly average all 80 to 100 results. Now, calculate the standard deviation for the number of heads obtained σ_h using the formula in the lab manual; again, you can use a trick to avoid explicitly calculating with all 80 to 100 results. If the trick is not obvious to you, discuss this with your classmates or with me. In the limit of an infinite total number of tosses, the average becomes 3 and the standard deviation σ_h becomes $1.2344\ldots$. The results you obtain for $\bar{h}$ and σ_h should be close to these numbers.

Finally, we note that the distribution as a function of h, the *number of heads*, is almost a Gaussian. Based on the average and the standard deviation obtained above, plot (with the other colored pencil) the Gaussian curve

$$C\,\mathrm{e}^{-(h-\bar{h})^2/(2\sigma_h^2)} \tag{1}$$

on your graph. (Don't forget to label the various things you have plotted.) Choose the constant C to be the total number of tosses divided by 3.09418; this should give you the correct normalization. How does the Gaussian you plotted agree with the expected number of tosses (note especially the regions $h \le 0$ and $h \ge 6$)? How does the agreement exemplify the central limit theorem? If you were to toss just one or two coins, would the resulting distribution be very Gaussian? How would you have to modify the experiment to get an exactly Gaussian distribution?

2 Units are your friend

1. It happens that there are about $\pi \cdot 10^7$ seconds in a year. Find the percentage error if I use this as the length of a year.

2. A car gets 22 miles per gallon of gasoline. Express 22 miles per gallon in terms of the units $1/\mathrm{cm}^2$. While this car is moving along, a passenger pours a can of gasoline from the car window onto the ground at the rate of 1 gallon per 22 miles. This creates a long, thin puddle of gasoline on the road. What is the cross sectional area, in square centimeters, of the puddle?

3. An oil slick on the surface of water spreads out until it is just one molecule thick. If a spherical drop of oil 2.0 mm in diameter spreads out into a circular oil slick 3.5 inches in diameter, estimate the length of a single oil molecule. The oil molecules are long and thin and they line up vertically.